On Two Thiagarajan's Conjectures

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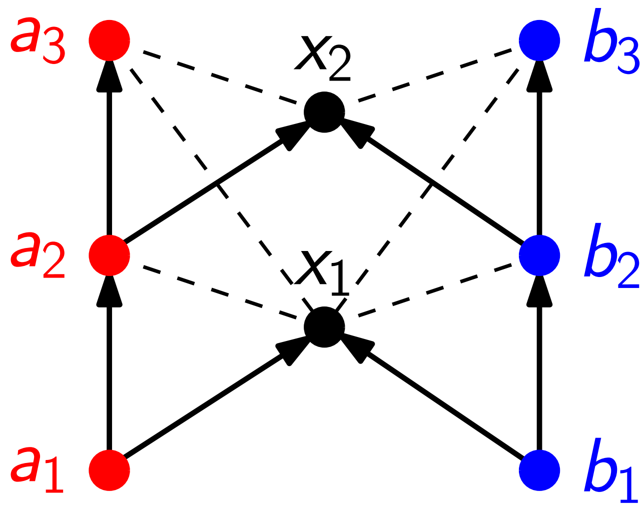
(Prime) Event Structures

An event structure is a triple $\mathcal{E} = (E, \leq, \#)$ where

- E is a set of events
- $\leq$ is a partial order on E
- $\#$ is a (binary) conflict relation on E
- $\downarrow e := \{e' \in E : e' \leq e\}$ is finite for any $e \in E$
- $e \# e'$ and $e' \leq e'' \implies e \# e''$



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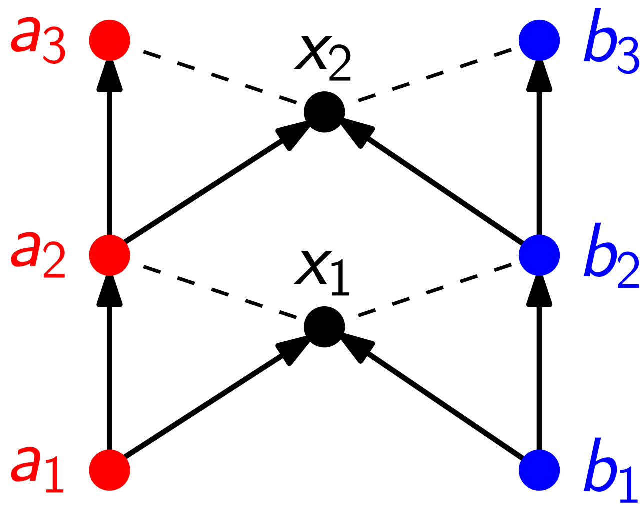
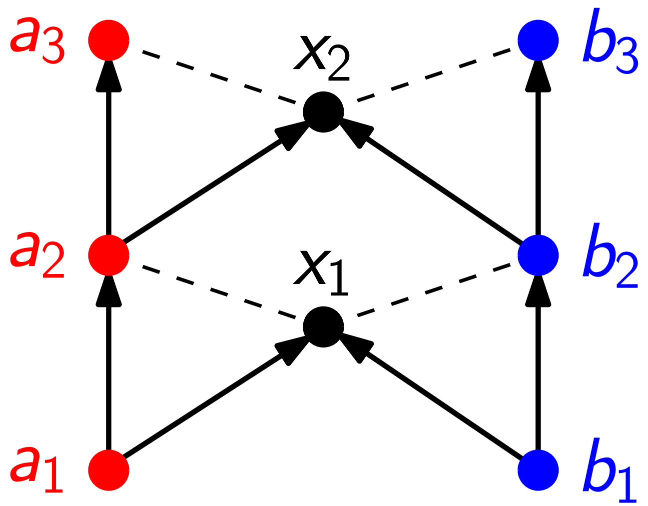


- e_1 and e_2 are in minimal conflict, $e_1 \#_\mu e_2$, if there is no event $e_1' \leq e_1$ such that $e_1' \# e_2$ (and vice versa)
- e_1 and e_2 are concurrent, $e_1 \| e_2$, if they are not comparable for $\leq$ and not in conflict

Configurations and Domains

A finite subset $c \subseteq E$ is a configuration if

- c is downward-closed: $e \in c$ and $e' \leq e \implies e' \in c$
- c is conflict-free: $e, e' \in c \implies (e, e') \notin \#$



- $\{a_1, a_2, b_1\}$ is a configuration
- $\{a_1, b_1, x_1\}$ is a configuration
- $\{a_1, a_2, b_2\}$ is not a configuration
- $\{a_1, a_2, b_1, x_1\}$ is not a configuration

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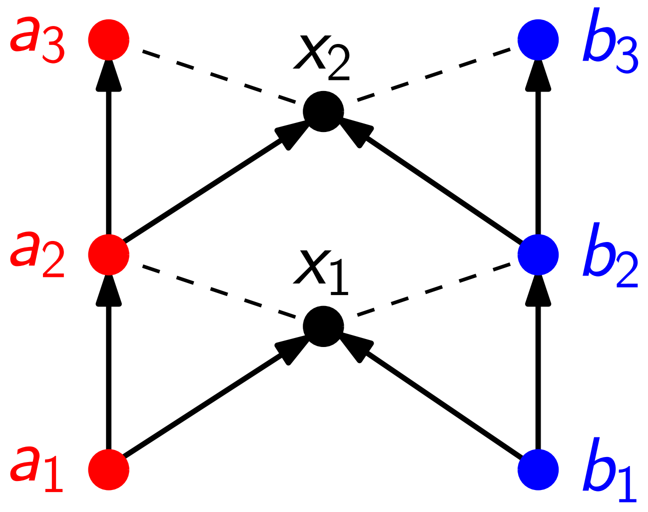
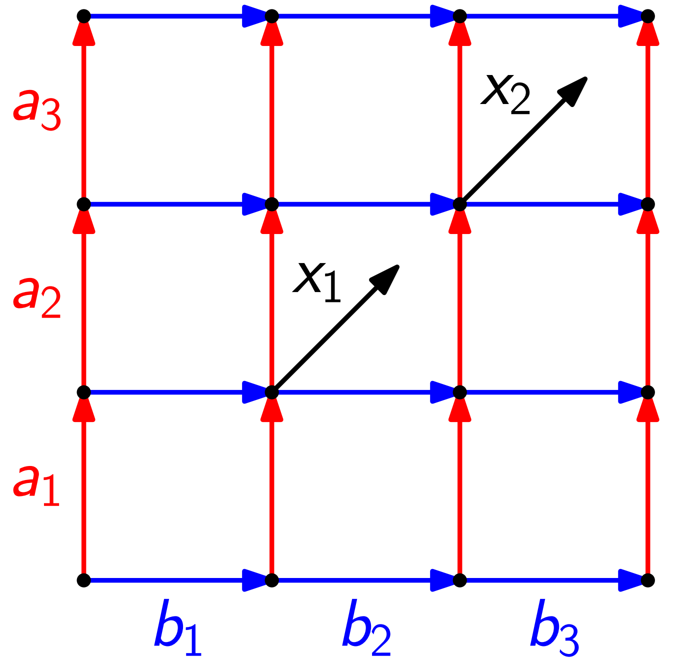
The domain $D(\mathcal{E})$ is a directed graph where

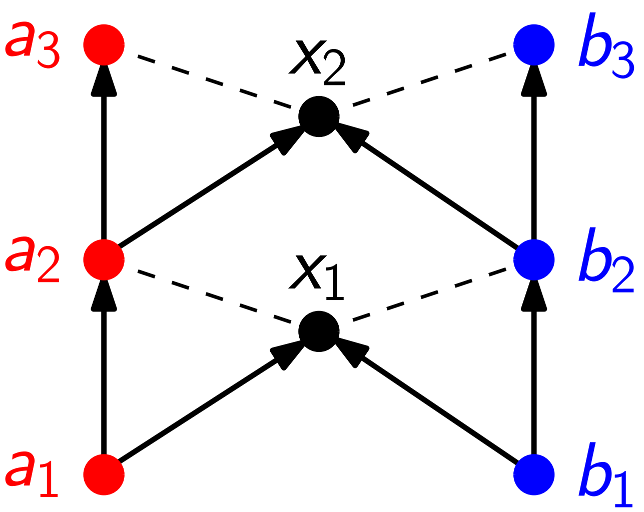
- the vertices of $D(\mathcal{E})$ are the configurations of $\mathcal{E}$
- $c \to c'$ if $c' = c \cup \{e\}$ for some event $e \notin c$

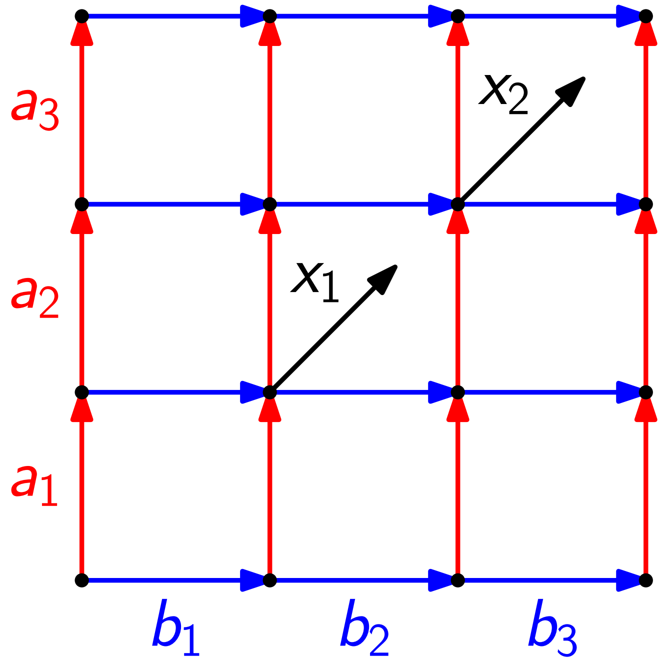




Labeled Event Structures

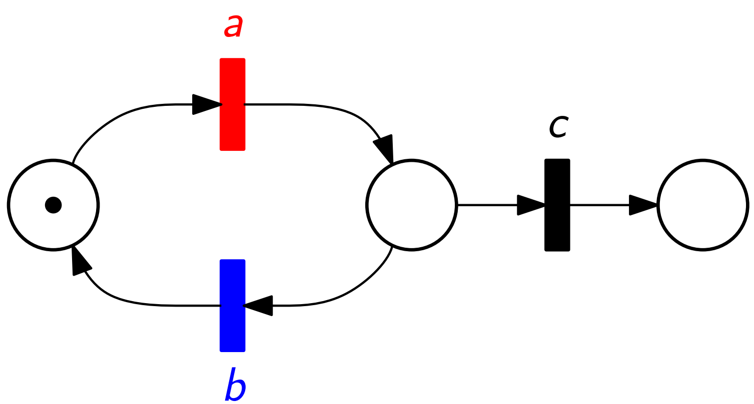
- A labeled event structure $(\mathcal{E}, \lambda)$ is an event structure $\mathcal{E}$ with a labeling $\lambda : E \to \Sigma$ (where Σ is a finite alphabet)
- λ is a nice labeling if $\lambda(e) \neq \lambda(e')$ when $e \| e'$ or $e \#_\mu e'$
- Equivalently, λ is a coloring of the edges of $D(\mathcal{E})$
  - Determinism: two edges with the same origin have distinct colors
  - Concurrency: two opposite edges of a square have the same color





Event Structures and 1-safe Petri Nets

To any finite 1-safe Petri Net N, one can associate an event structure $\mathcal{E}_N$ with a nice labeling λ_N



A 1-safe Petri Net is $N = (S, \Sigma, F, m_0)$

- S: places
- Σ: transitions
- $F \subseteq (S \times \Sigma) \cup (\Sigma \times S)$: flow relation
- $m_0 \subseteq S$: initial marking

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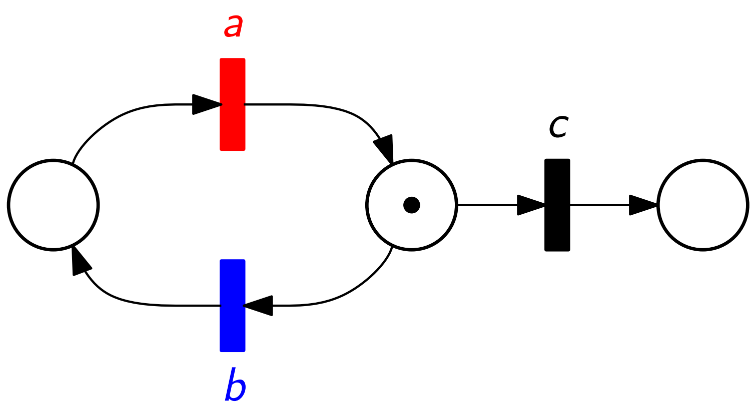

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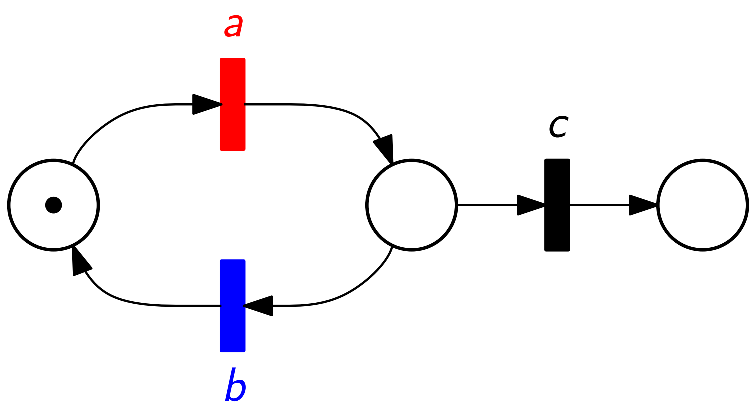

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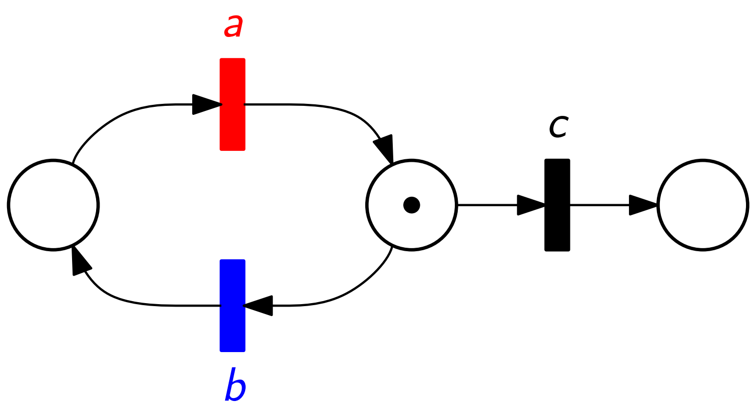

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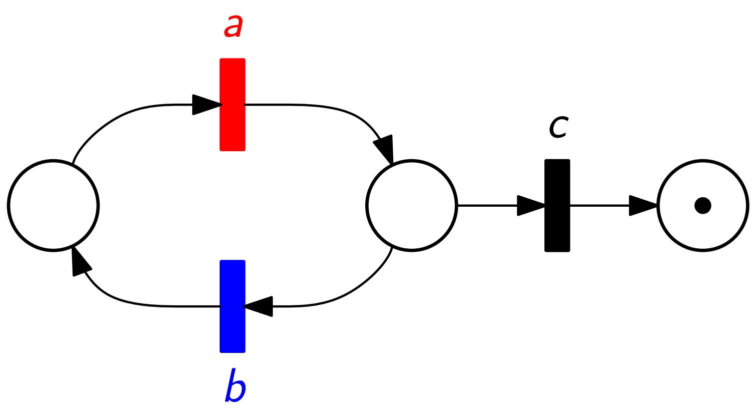

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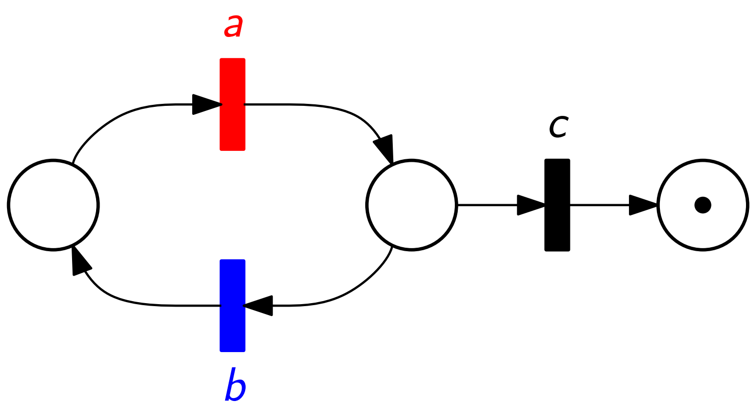

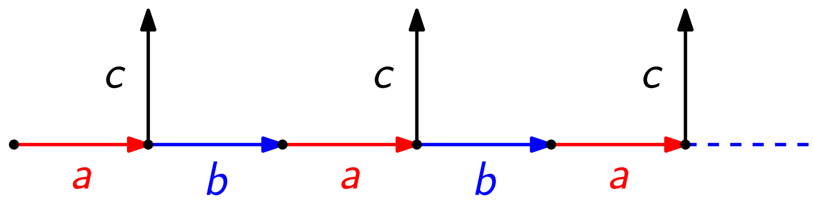

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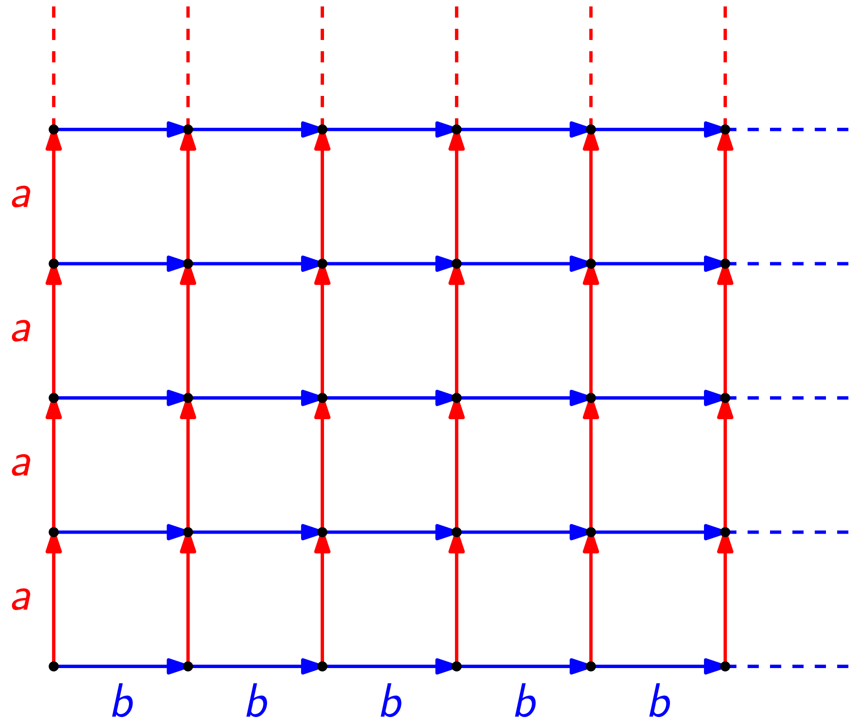

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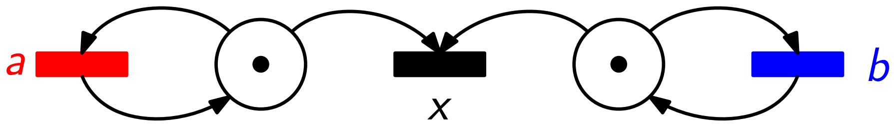

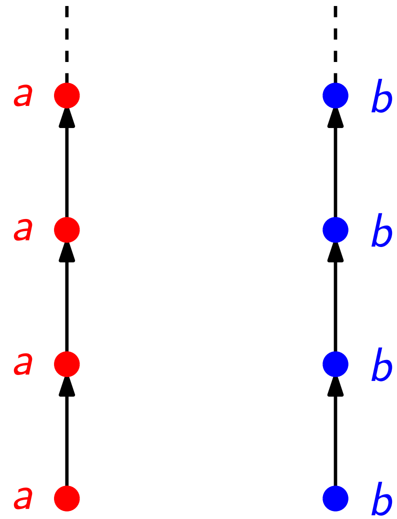

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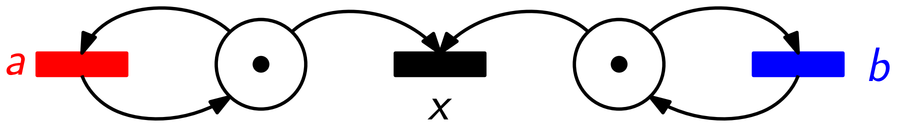

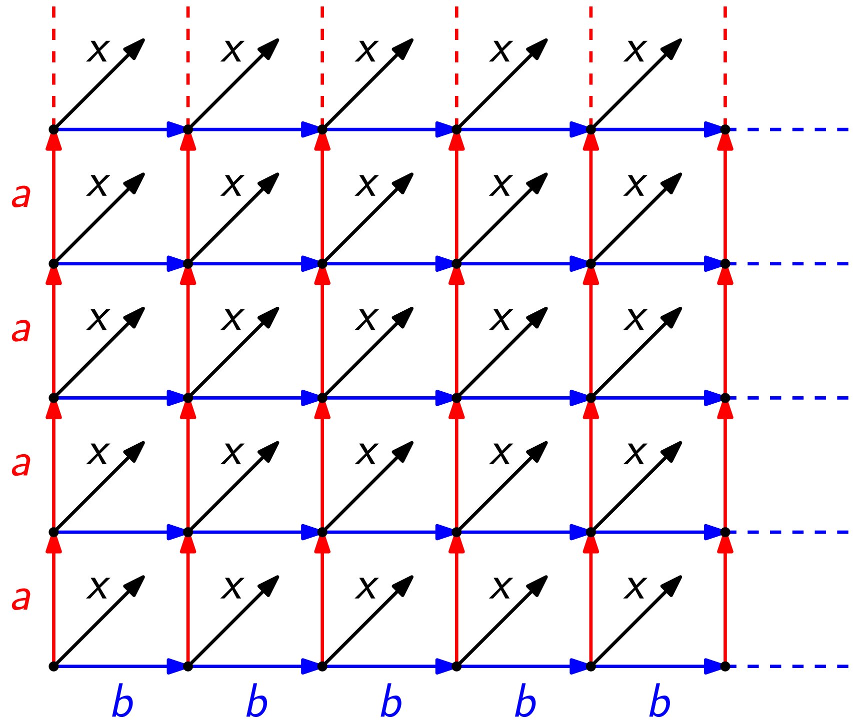

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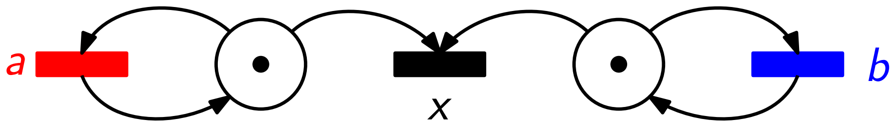
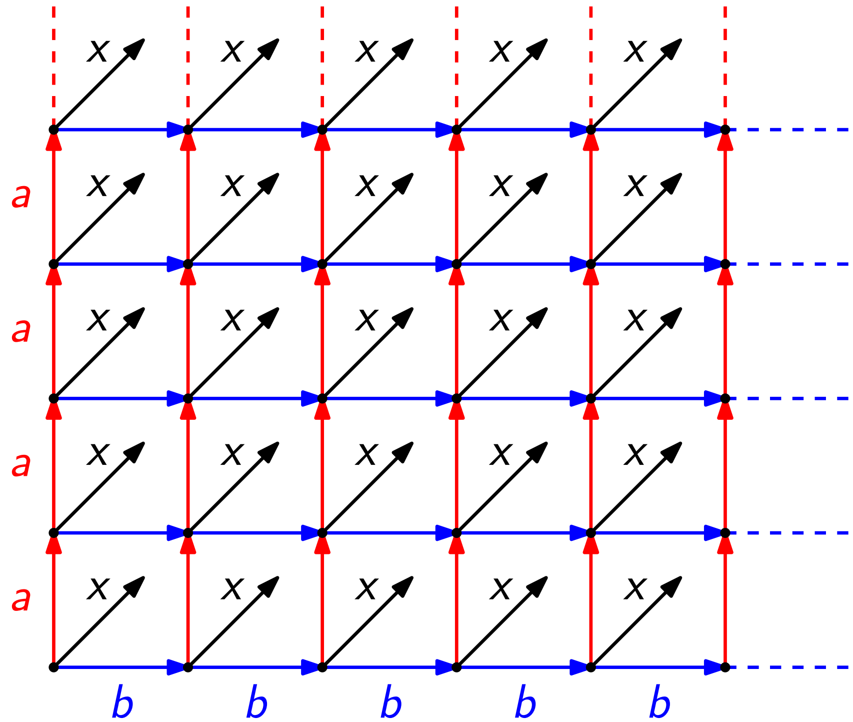
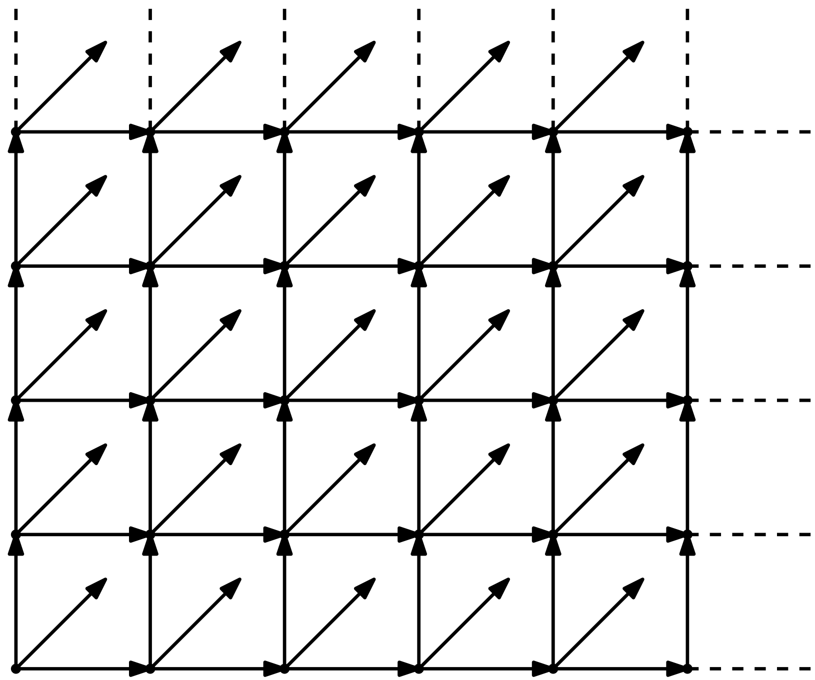




There is some regularity in the event structures arising from 1-safe Petri Nets

Regular Event Structures

- In $D(\mathcal{E})$, the future of a configuration c is the subgraph induced by the configurations reachable from c in $D(\mathcal{E})$
- Two configurations c, c' are equivalent, $cR_{\mathcal{E}}c'$, if they have isomorphic futures



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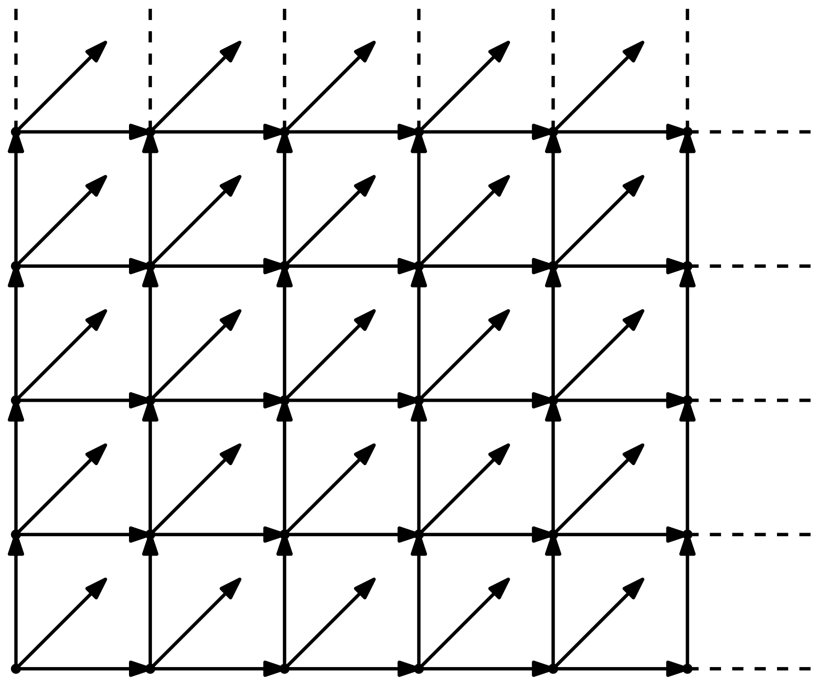
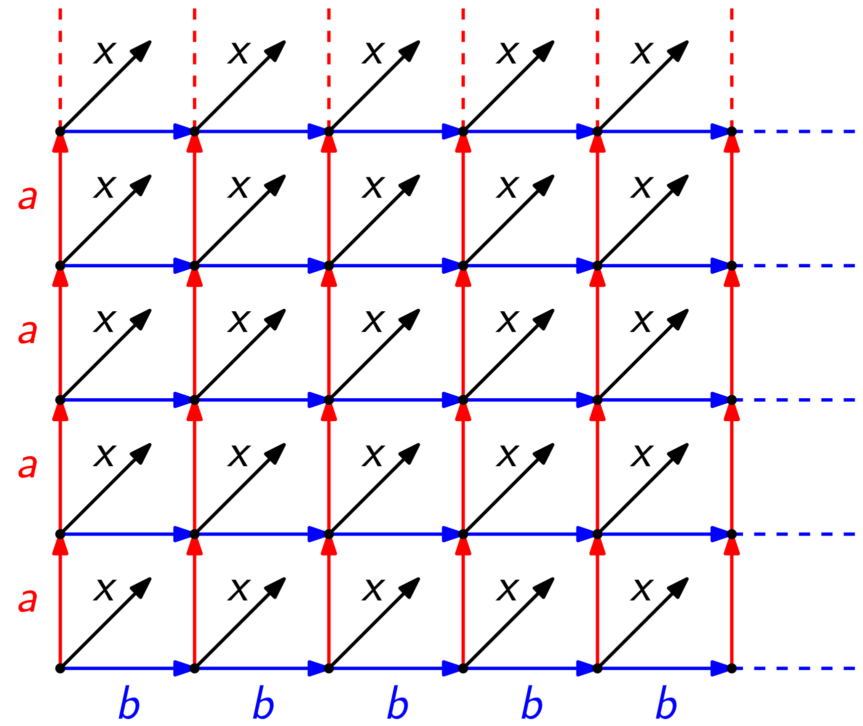
- Two configurations c, c' are equivalent, $cR_{\mathcal{E}}c'$, if they have isomorphic futures
- A event structure $\mathcal{E}$ is regular if $D(\mathcal{E})$ has a finite degree and $R_{\mathcal{E}}$ has a finite number of equivalence classes



Regular Labeled Event Structures

If $(\mathcal{E}, \lambda)$ is a labeled event structure

- Two configurations c, c' are equivalent, $cR_{\mathcal{E}}c'$, if they have isomorphic labeled futures
- $(\mathcal{E}, \lambda)$ is regular if λ is a nice labeling and $R_{\mathcal{E}}$ has a finite number of equivalence classes
- We say that λ is a regular nice labeling of $\mathcal{E}$



Event Structures and 1-safe Petri Nets

Any finite 1-safe Petri net gives a regular labeled event structure (and some extra properties)

Theorem [Thiagarajan '96 (+ Morin '05)]

Any regular labeled event structure $(\mathcal{E}, \lambda)$ is isomorphic to the event structure arising from a 1-safe Petri Net

Thiagarajan's regularity conjecture '96

Any regular event structure $\mathcal{E}$ is isomorphic to the event structure arising from a 1-safe Petri Net

- True when $\mathcal{E}$ is conflict-free [Nielsen, Thiagarajan '02]
- True when the domain of $\mathcal{E}$ is context-free [Badouel, Darondeau, Raoult '99]

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Any regular event structure $\mathcal{E}$ is isomorphic to the event structure arising from a 1-safe Petri Net

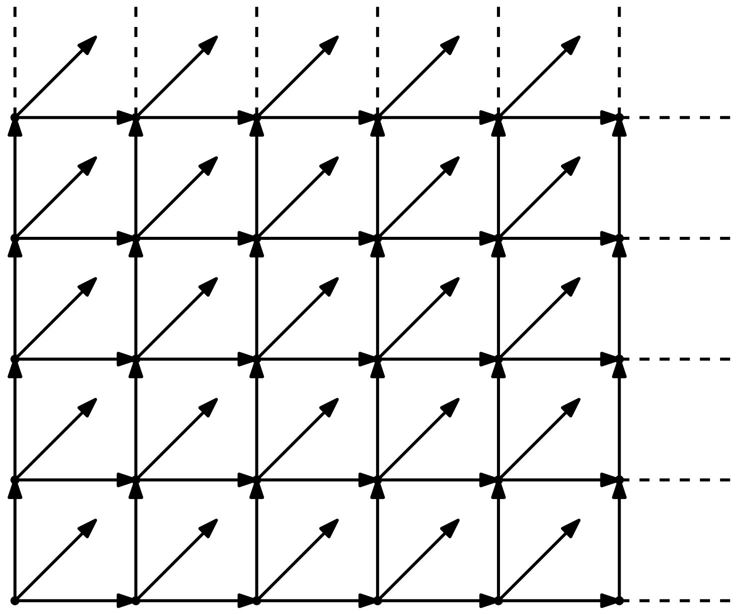
An equivalent condition

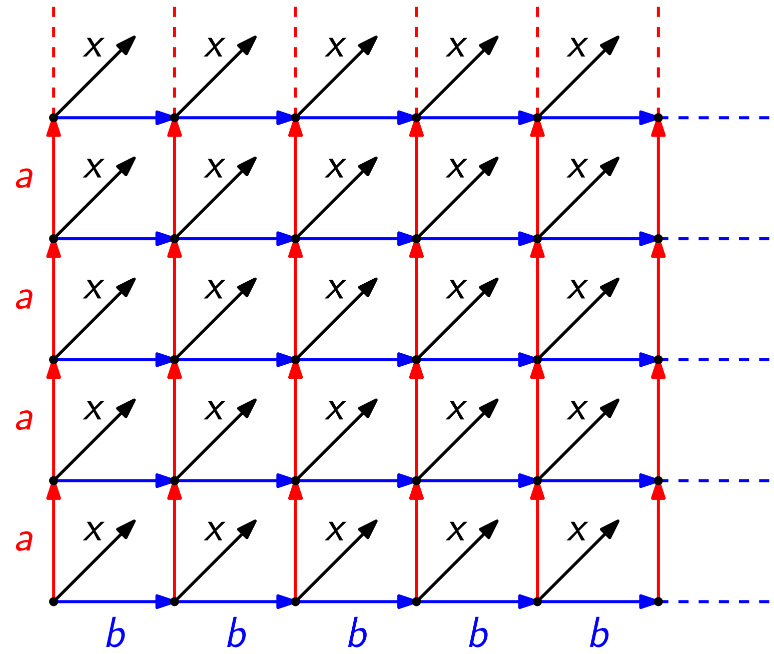
Any regular event structure $\mathcal{E}$ admits a regular nice labeling

The Problem

Our Question

Given a regular event structure $\mathcal{E}$, can we always find a regular nice labeling of $\mathcal{E}$?

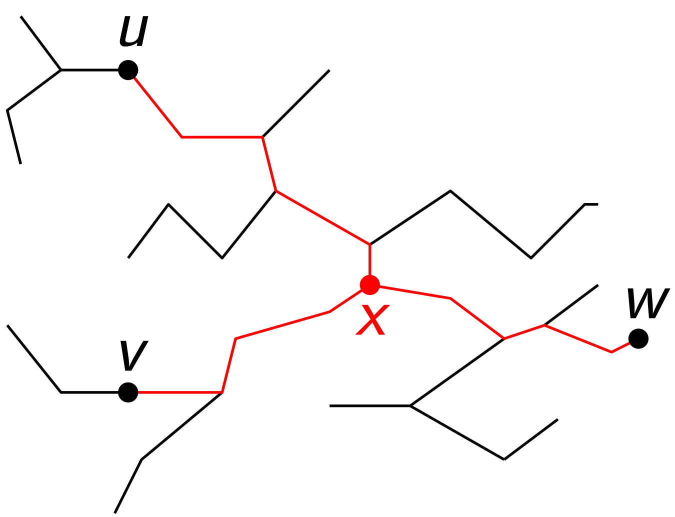




Median graphs

Definition

A graph $G = (V, E)$ is median if for all $u, v, w \in V$, there exists a unique $x \in V$ lying on a (u, v)-shortest path, a (u, w)-shortest path, and a (v, w)-shortest path



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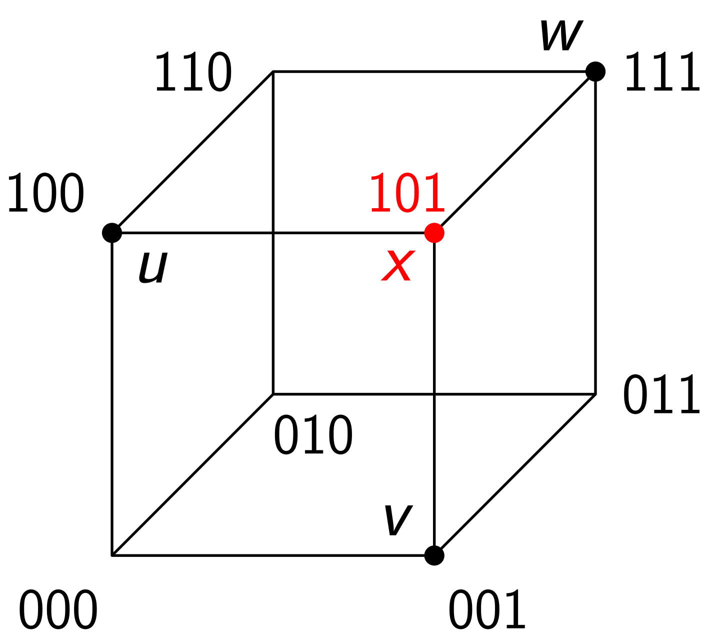

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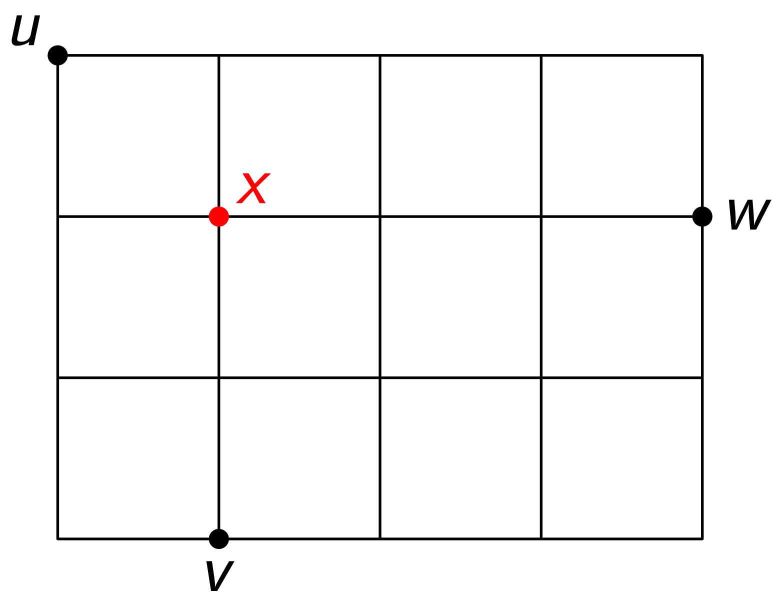

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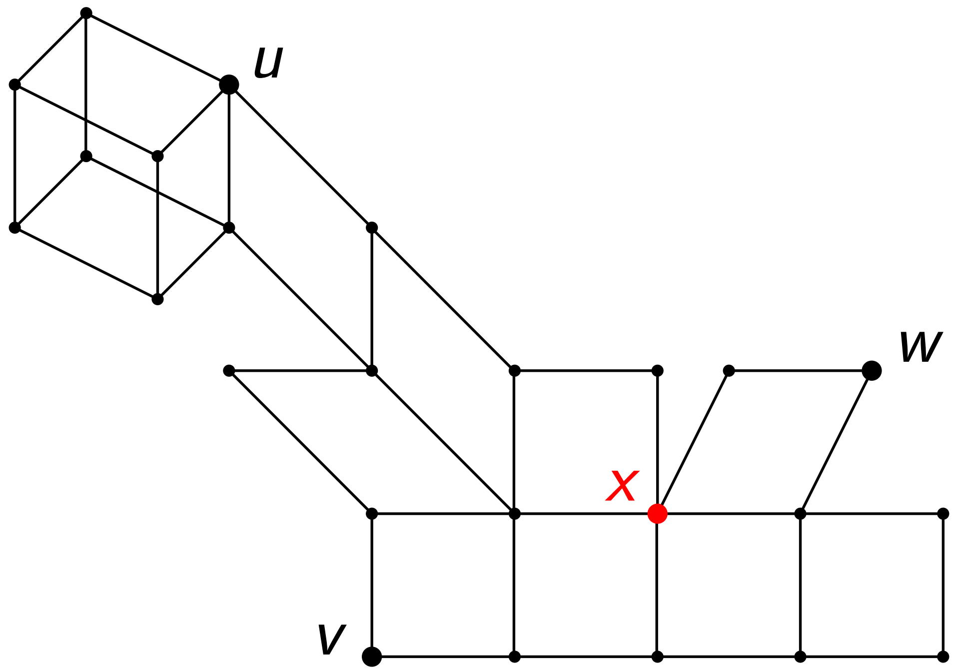
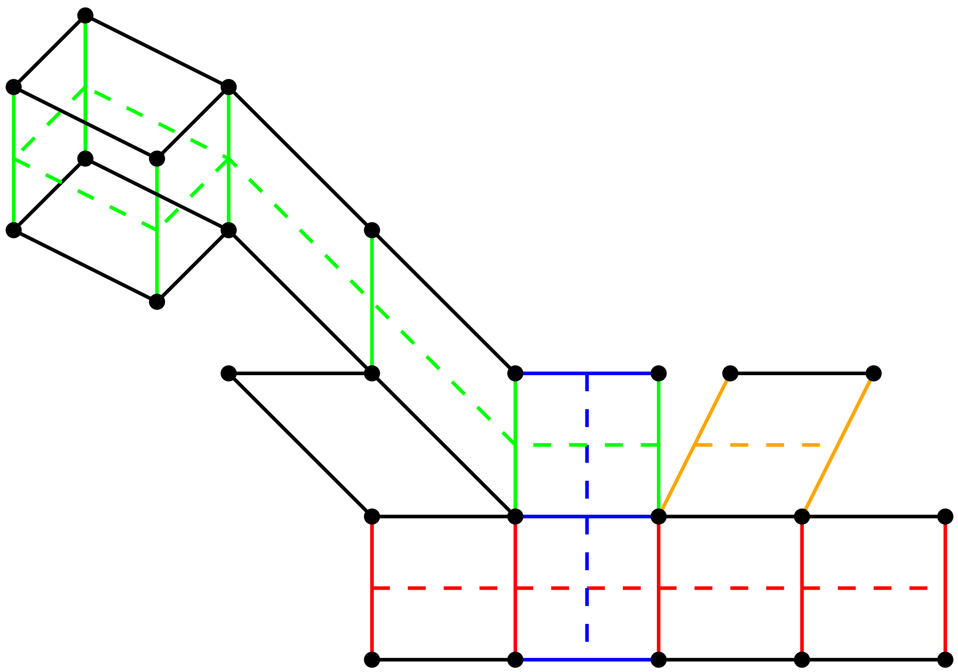


Hyperplanes [Sageev]

In a median graph G, the Djoković-Winkler relation Θ is defined as follows:

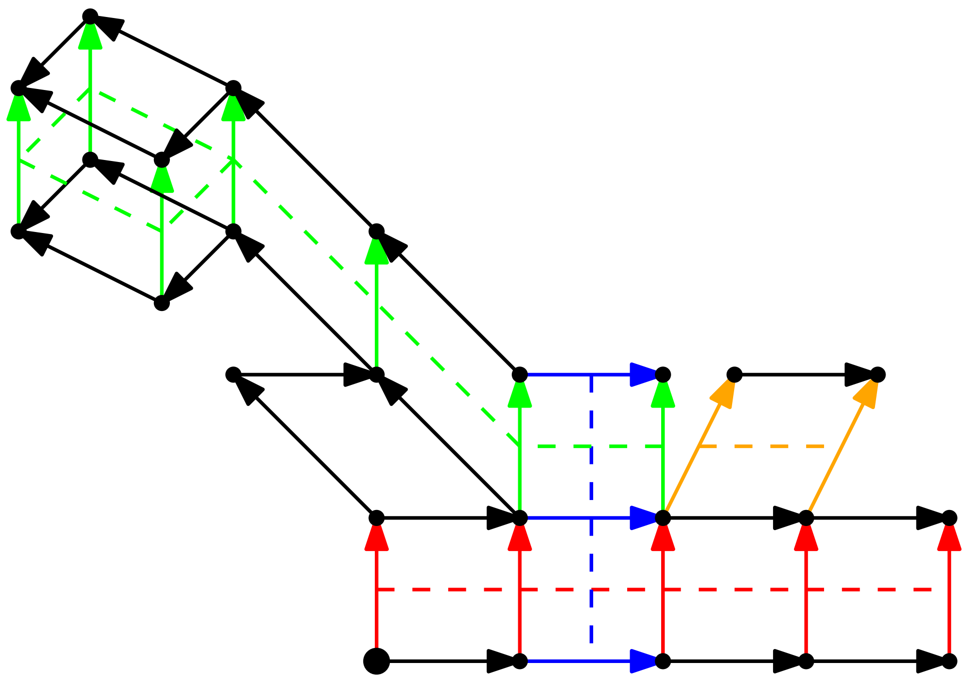
- $e_1 \Theta_1 e_2$ if e_1 and e_2 are two two opposite edges of a square
- $\Theta = \Theta_1^*$
- an hyperplane of G is an equivalence class of Θ



Median Graphs and Event Structures

Theorem [Barthélémy and Constantin '93]

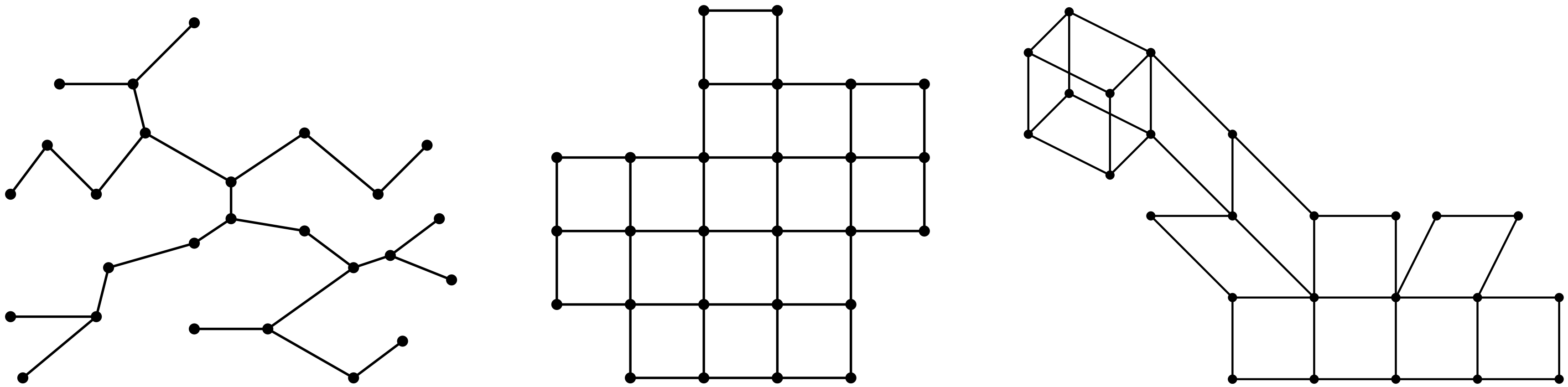
- $D(\mathcal{E})$ is a median graph (forgetting the orientation)
- Any pointed median graph is the domain of an event structure



CAT(0) cube complexes

A cube complex is a cell complex where each cell is a cube and when two cubes intersect, they intersect on a common face.

The 1-skeleton of X is the underlying graph $(V(X), E(X))$



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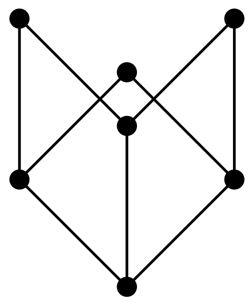
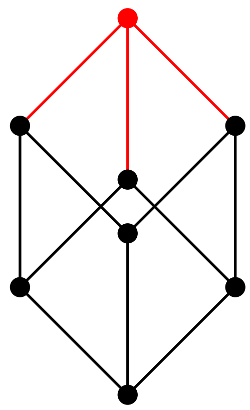
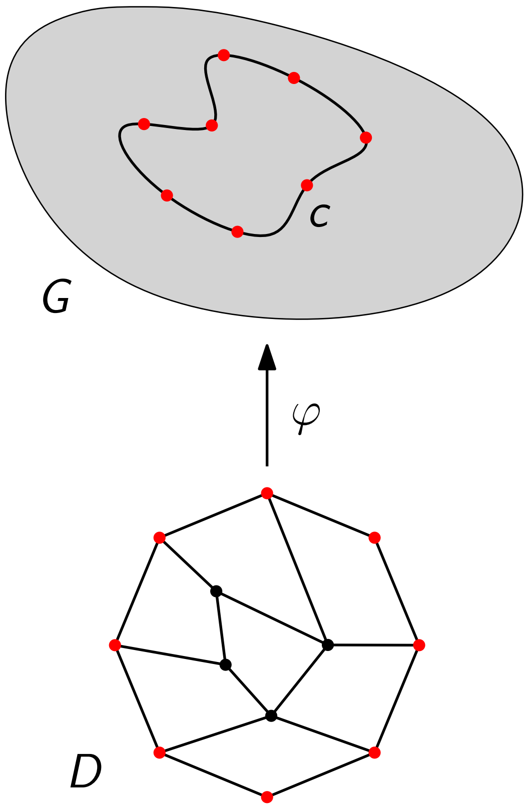
A cube complex X is CAT(0) if

- X is nonpositively curved (NPC) [Gromov]
- X is simply connected







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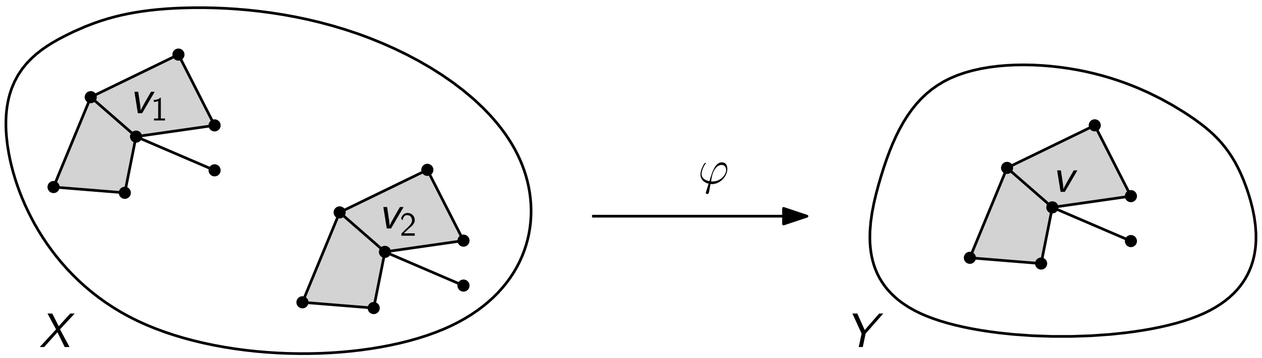
Theorem [Chepoi '00]

Median graphs are exactly the 1-skeletons of CAT(0) cube complexes

Covers of cube complexes

A cube complex X is a cover of the cube complex Y if there is a simplicial map $\varphi : V(X) \to V(Y)$ that is locally bijective



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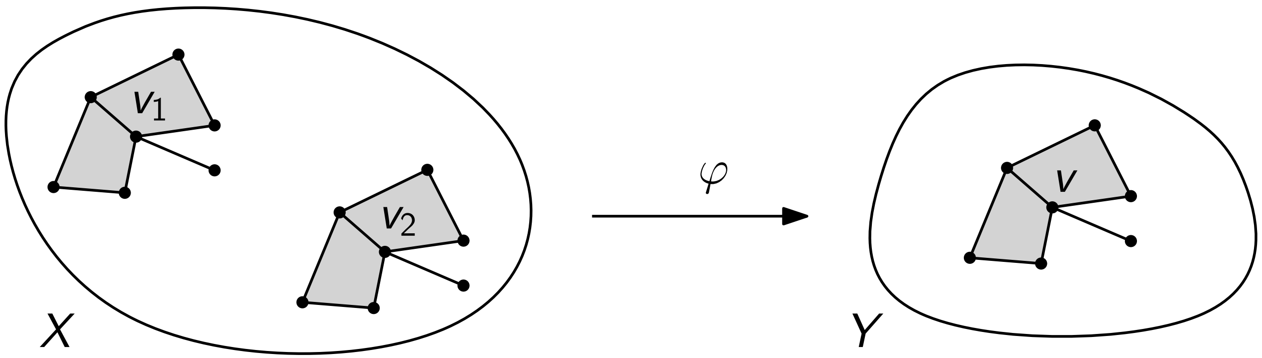

Theorem (from Topology)

- *Any complex X has a universal cover $\widetilde{X}$ such that if Y is a cover of X then $\widetilde{X}$ is a cover of Y*
- *X is simply connected if and only if $\widetilde{X} = X$*

Constructing Event Structures from NPC complexes

Recall that a cube complex is Non Positively Curved (NPC) if it satisfies Gromov's cube condition

- Starting from a finite NPC cube complex X, its universal cover $\widetilde{X}$ is a CAT(0) cube complex
- We have a finite number of equivalence classes of vertices in $\widetilde{X}$ up to isomorphism

Problem

We need to have some orientations on the edges to get the domain of an event structure

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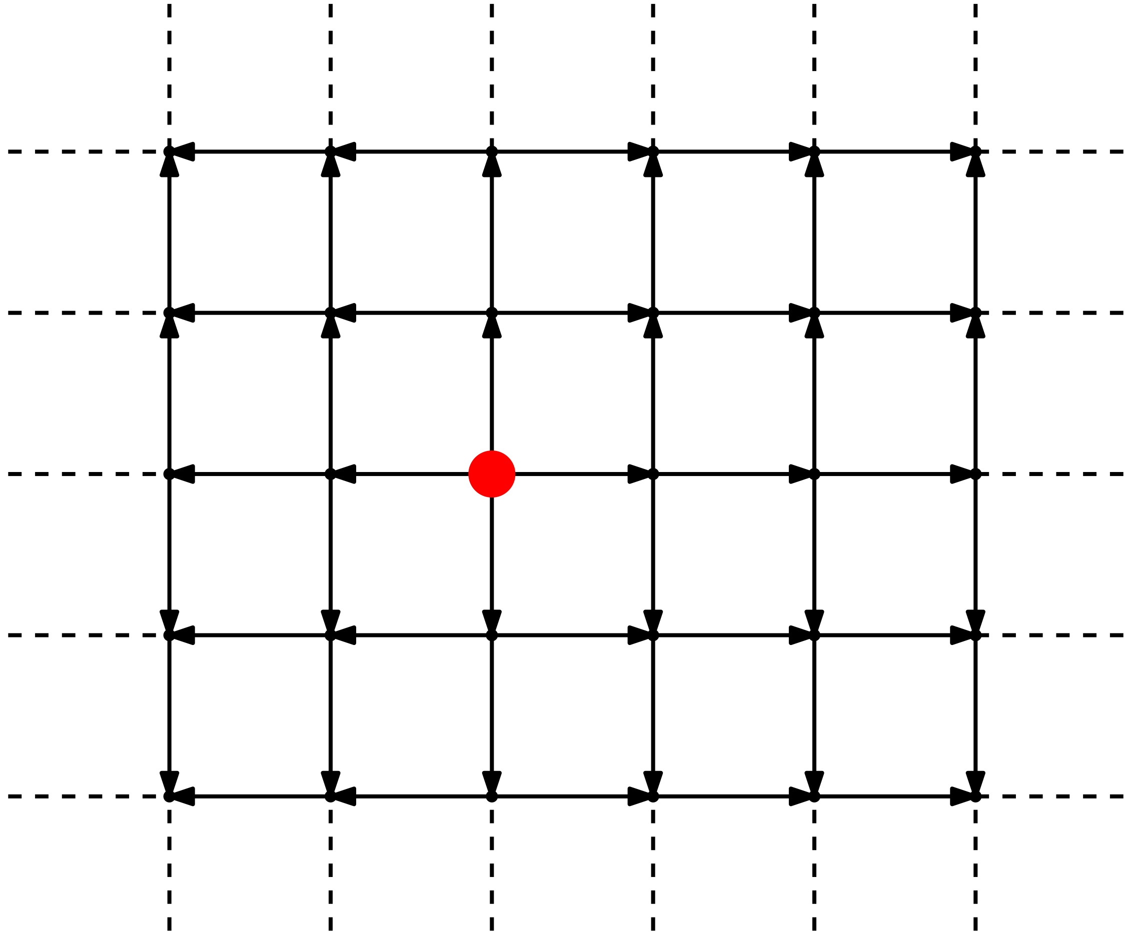
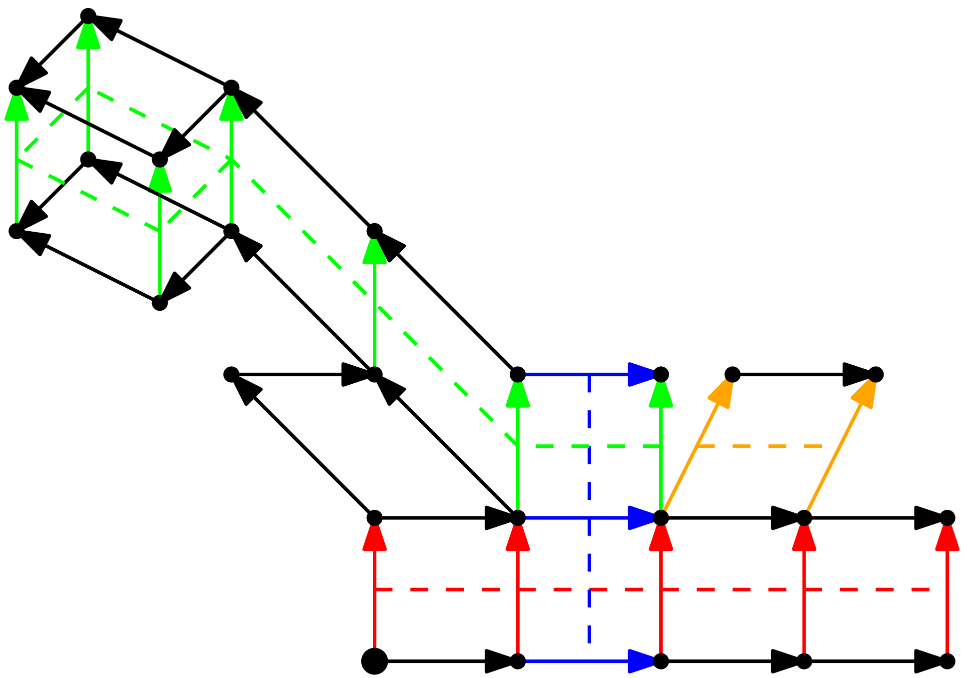


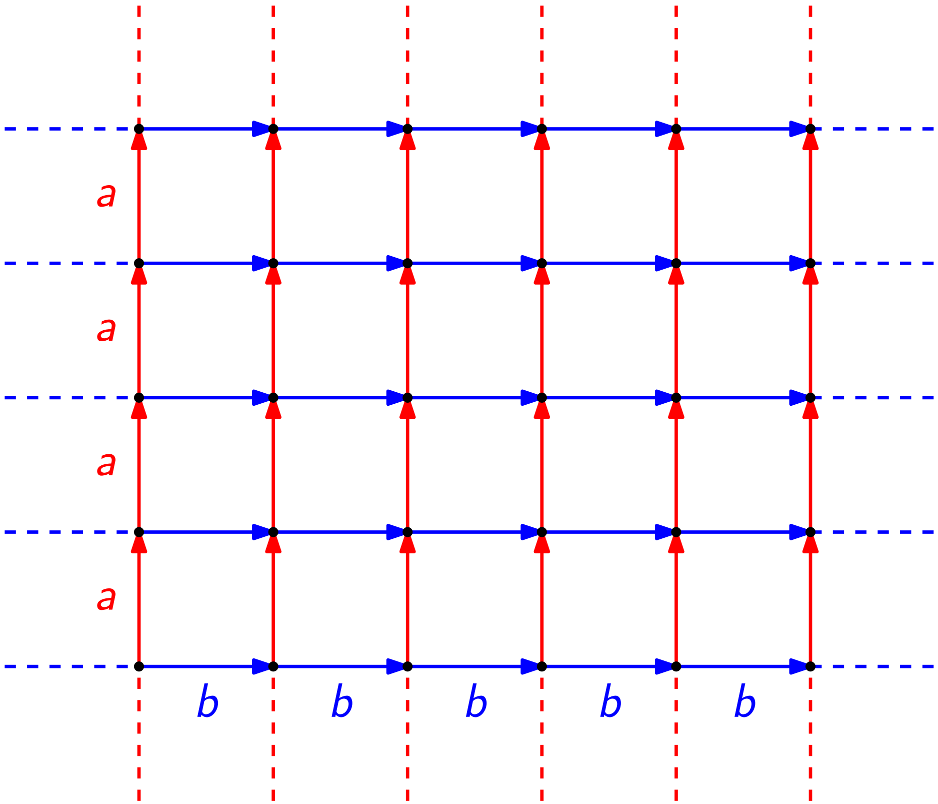
Directed NPC complexes

A directed NPC complex is a complex such that each edge is directed in such a way that two opposite edges of a square have the same direction



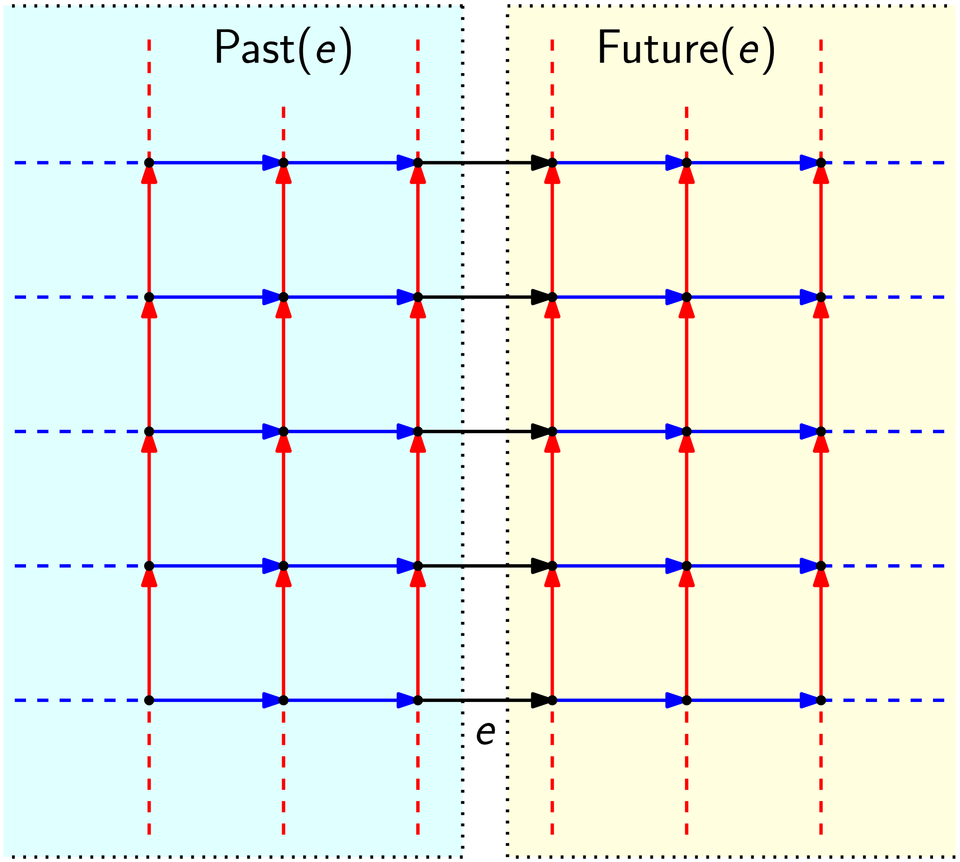
From Directed NPC complexes to Event Structures

- Starting from a finite directed NPC complex X, we construct its universal cover $\widetilde{X}$
- We have a finite number of classes of futures
- But vertices can have an infinite past . . .



Cutting along Hyperplanes

- In $\widetilde{X}$, edges belonging to the same hyperplane have the same orientation
- In a CAT(0) cube complex, hyperplanes are separators
  - For each hyperplane e, we define $\mathsf{Past}(e)$ and $\mathsf{Future}(e)$



Cutting along Hyperplanes

- In $\widetilde{X}$, edges belonging to the same hyperplane have the same orientation
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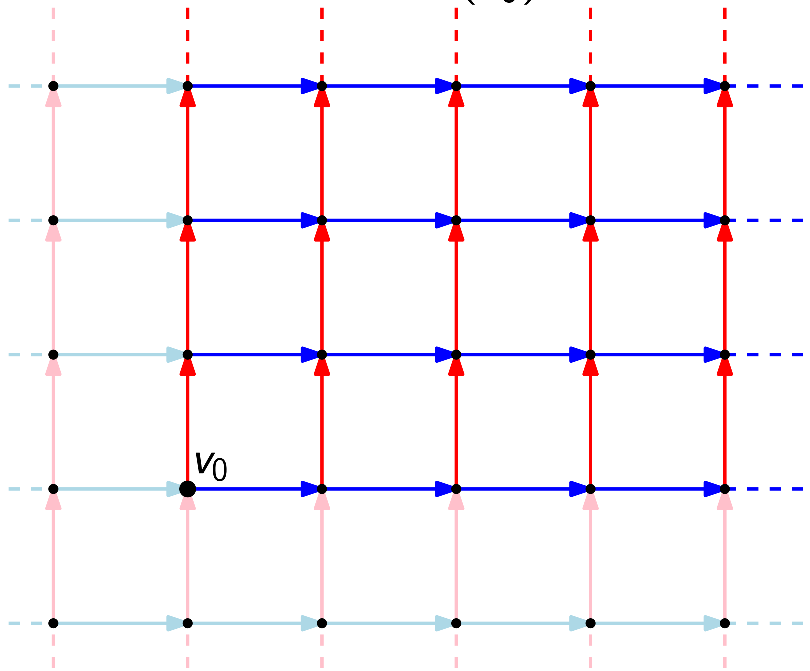
- Pick $v_0 \in \widetilde{X}$, let $\mathsf{Past}(v_0) = \{e \mid v_0 \in \mathsf{Future}(e)\}$ and

$$\widetilde{X}_{v_0} = \bigcap_{e \in \mathsf{Past}(v_0)} \mathsf{Future}(e)$$



Cutting along Hyperplanes

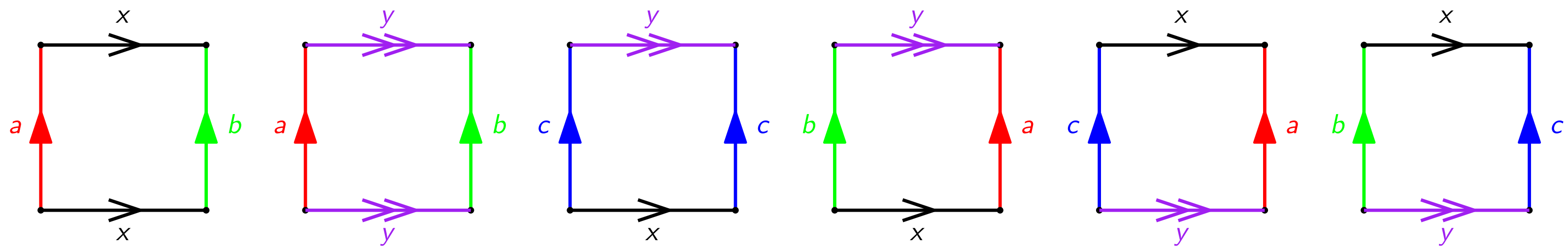
- In $\widetilde{X}$, edges belonging to the same hyperplane have the same orientation
- In a CAT(0) cube complex, hyperplanes are separators
- Pick $v_0 \in \widetilde{X}$, let $\mathsf{Past}(v_0) = \{e \mid v_0 \in \mathsf{Future}(e)\}$ and

$$\widetilde{X}_{v_0} = \bigcap_{e \in \mathsf{Past}(v_0)} \mathsf{Future}(e)$$

- Starting from a finite directed NPC complex X, we have constructed a pointed CAT(0) cube complex $\widetilde{X}_{v_0}$, i.e., the domain of an event structure
- The number of classes of futures is bounded by $|V(X)|$
- $\widetilde{X}_{v_0}$ is the domain of a regular event structure

Wise's directed NPC complex X

A colored directed NPC complex with 1 vertex, 2 "horizontal" edges (x and y), 3 "vertical" edges (a, b, and c), 6 squares



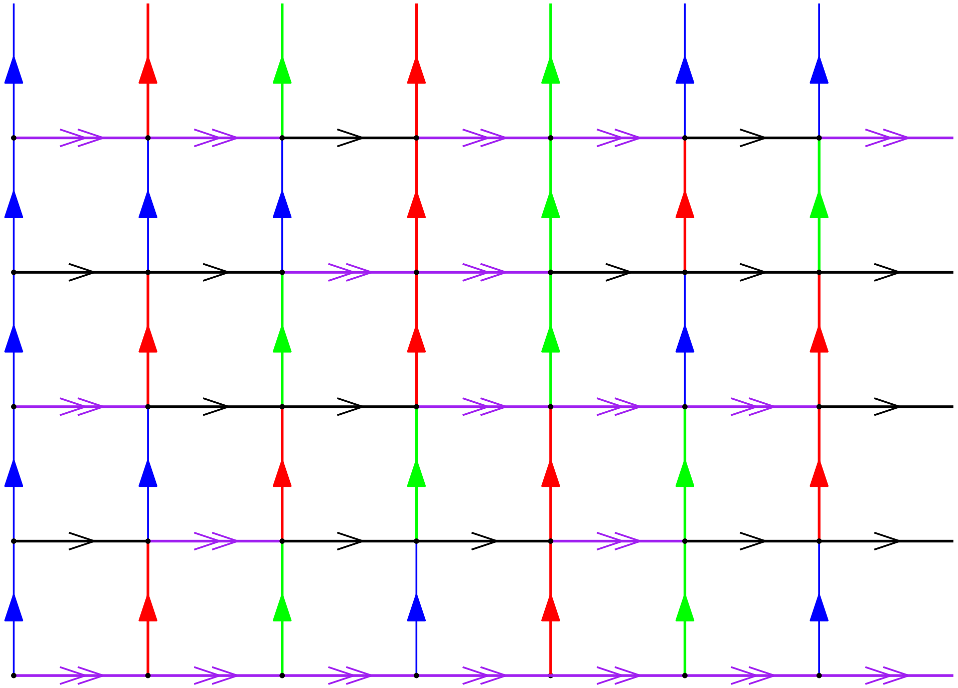
- it defines a square complex
- it is directed non positively curved

Warning!!

Colors have nothing to do with the labels of an event structure

An aperiodic tiling in the universal cover $\widetilde{X}$ of X

In the universal cover $\widetilde{X}$ of X, the quarter of plane defined by y^ω and c^ω is aperiodic

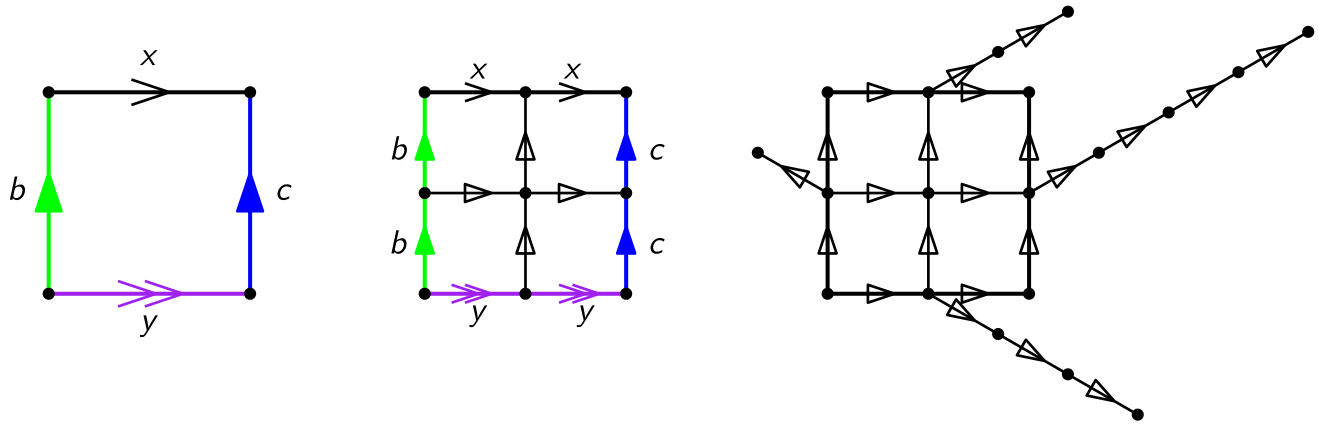


Proposition [Wise '96]

All horizontal words starting on the side of the quarter of plane are distinct

From $\widetilde{X}$ to a colorless domain $\widetilde{W}_v$

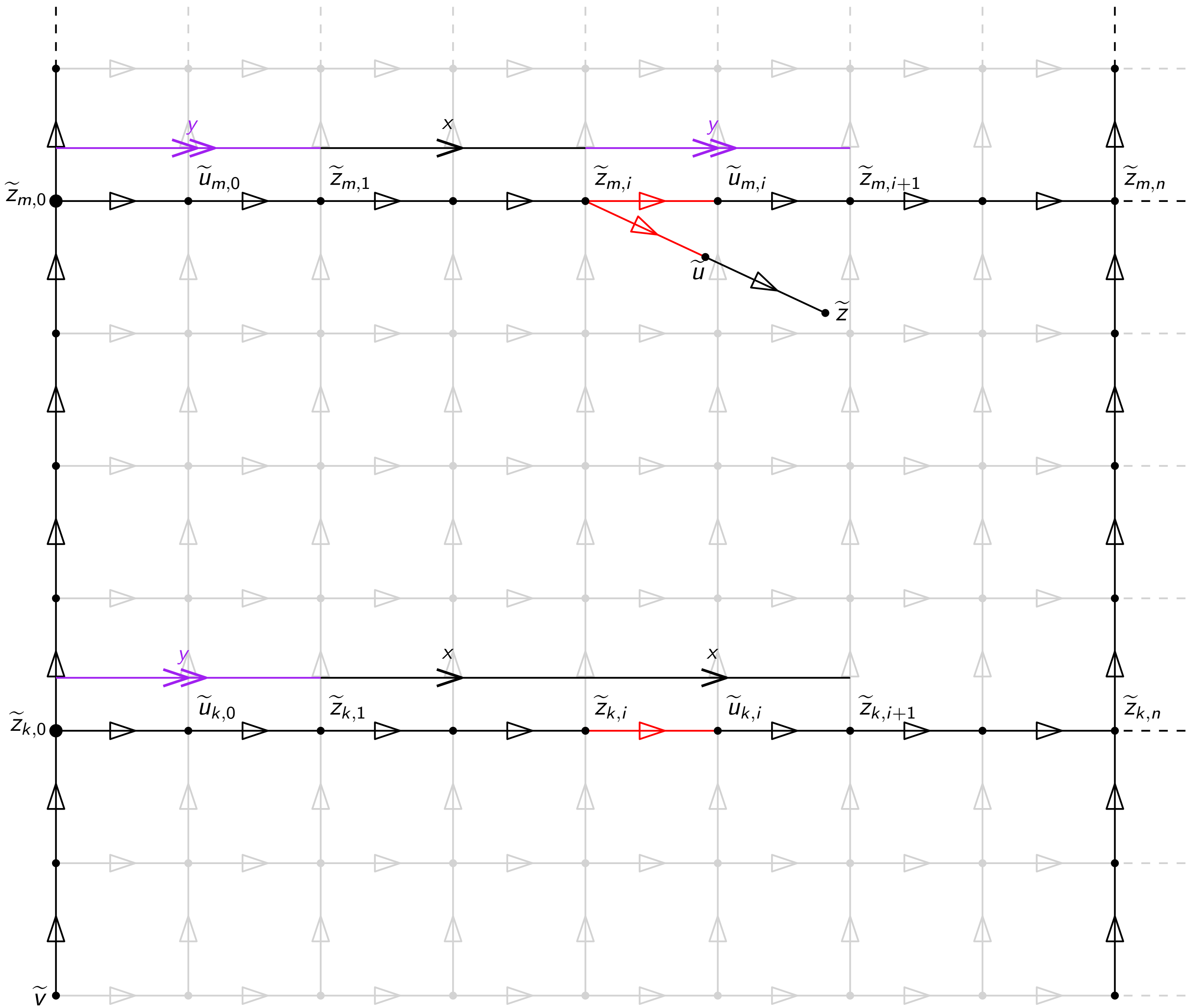
We encode the colors of the edges by a trick



In X, each color is “replaced” by a directed path attached to the “middle” of the edge

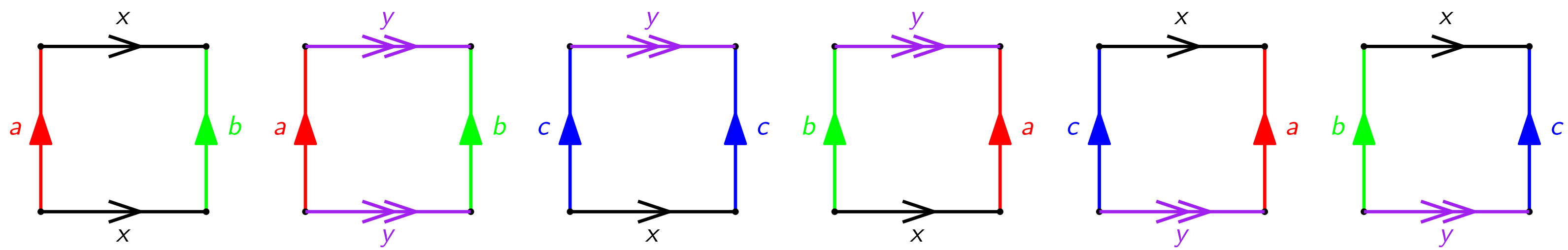
Let W be the colorless directed NPC complex obtained
Consider its universal cover $\widetilde{W}$
Pick a vertex v in $\widetilde{W}$ and consider the domain $\widetilde{W}_v$

$\widetilde{W}_v$ has no regular nice labeling



Counterexamples arise from aperiodic tilesets

Wise's complex is obtained from a 4-way deterministic tileset



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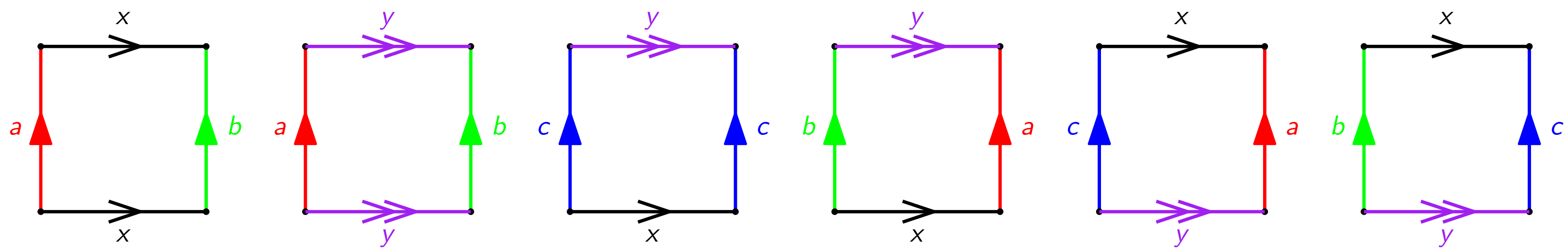


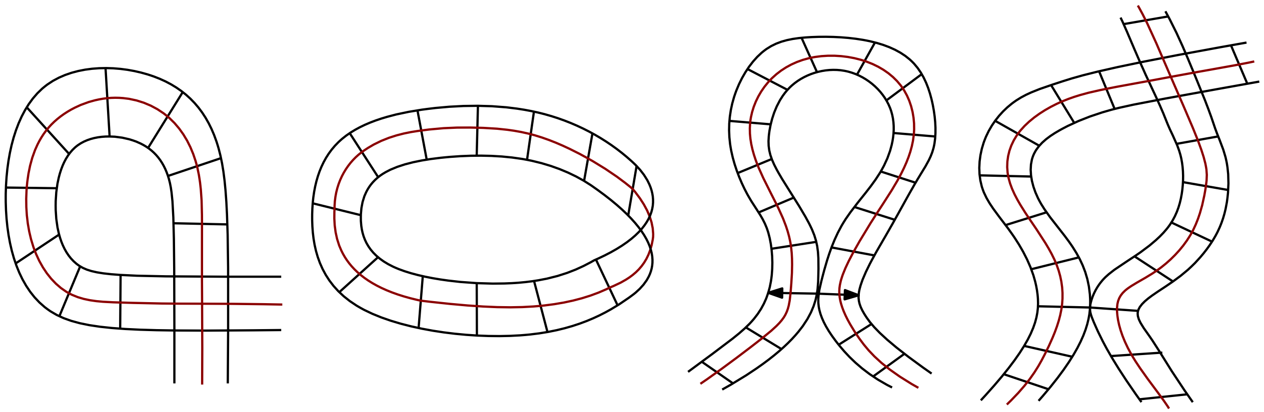
Any aperiodic 4-way deterministic tileset gives a counterexample to Thiagarajan's conjecture

Theorem

- There exists a 4-way deterministic aperiodic tileset [Kari, Papasoglu '99]
- Deciding if a 4-way deterministic tileset tiles the plane is undecidable [Lukkarila '09]

On the positive side: special cube complexes

A NPC complex is special if its hyperplanes behave nicely
[Haglund, Wise '08]



(a) no self-intersection
(b) no 1-sided hyperplane
(c) no direct self-osculation
(d) no interosculation

A finite NPC complex is virtually special if it has a finite cover that is special

1-safe Petri nets and special cube complexes

Theorem

An event structure $\mathcal{E}$ admits a regular nice labeling

$\Leftrightarrow$ *$\mathcal{E}$ is isomorphic to the event structure arising from a 1-safe Petri Net* *[Thiagarajan '96]*

$\Leftrightarrow$ *there exists a finite directed (virtually) special cube complex X such that $D(\mathcal{E}) \simeq \widetilde{X}_v$*

MSO on Trace Regular Event Structures

Given a regular trace event structure $\mathcal{E}_N = (E, \leq, \#)$ with a regular trace labeling $\lambda : E \to \Sigma$, the MSO theory of $\mathcal{E}_N$ is defined by:

- first-order variables $x, y, \ldots$ representing events of $\mathcal{E}_N$
- second-order variables $X, Y, \ldots$ representing sets of events of $\mathcal{E}_N$
- atomic propositions $R_a(x)$ $(a \in \Sigma)$, $x \leq y$, $x \in X$
- boolean connectors $\neg, \wedge$ and quantifiers $\exists$

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One can express also

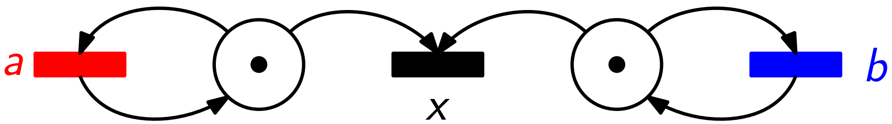
- $\vee, \Rightarrow, \Leftrightarrow, \forall, \subseteq, \ldots$ (as usual)
- the conflict $\#$ and the concurrency $\parallel$ relations
- the fact that a set is a configuration

When is MSO($\mathcal{E}_N$) decidable?

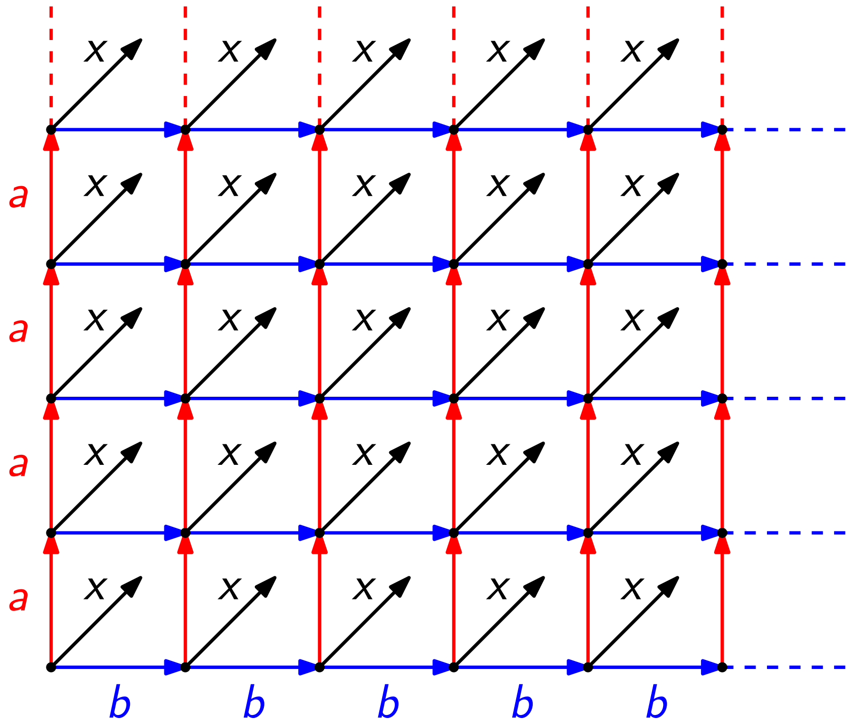
Question

Given a regular trace event structure $\mathcal{E}_N = (E, \leq, \#, \lambda)$, and an MSO sentence φ, can we decide if $\mathcal{E}_N \models \varphi$?

Not always [Walukiewicz]



One can encode problems that are undecidable on a grid



Thiagarajan's MSO conjecture

$\mathcal{E} = (E, \leq, \#)$ is grid free if there is no dijoint sets $X, Y, Z \subseteq E$ such that

- $X = \{x_0, \ldots, x_n\}$ is infinite with $x_0 < x_1 < \ldots$
- $Y = \{y_0, \ldots, y_n\}$ is infinite with $y_0 < y_1 < \ldots$
- $\forall x_i \in X, y_j \in Y, x_i \| y_j$
- there is a bijection $g : X \times Y \to Z$ such that if $z = g(x_i, y_j)$
  - $\forall i', x_{i'} \leq z$ iff $i' \leq i$
  - $\forall j', y_{j'} \leq z$ iff $j' \leq j$

Thiagarajan's MSO Conjecture '14

Given a regular trace event structure $\mathcal{E}_N = (E, \leq, \#, \lambda)$, the MSO($\mathcal{E}_N$) is decidable iff $\mathcal{E}_N$ is grid-free

Hyperbolic median graphs

Proposition [Folklore]

A median graph G is hyperbolic iff isometric square grids of G are bounded

Proposition

For a regular event structure $\mathcal{E} = (E, \leq, \#)$, $D(\mathcal{E})$ is hyperbolic iff there is no disjoint conflict-free infinite sets $X, Y \subseteq E$ such that $\forall x \in X, y \in Y$, $x \| y$

Corollary

If $D(\mathcal{E})$ is hyperbolic, then $\mathcal{E}$ is grid-free

The MSO logic of the domains MSO($D(\mathcal{E})$)

Given a trace regular event structure $\mathcal{E} = (E, \leq, \#, \lambda)$, $D(\mathcal{E})$ is a directed labeled digraph $(V, (E_a)_{a \in \Sigma})$

MSO($D(\mathcal{E})$))

- first-order variables $x, y, \ldots$ representing vertices of $D(\mathcal{E})$
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Proposition

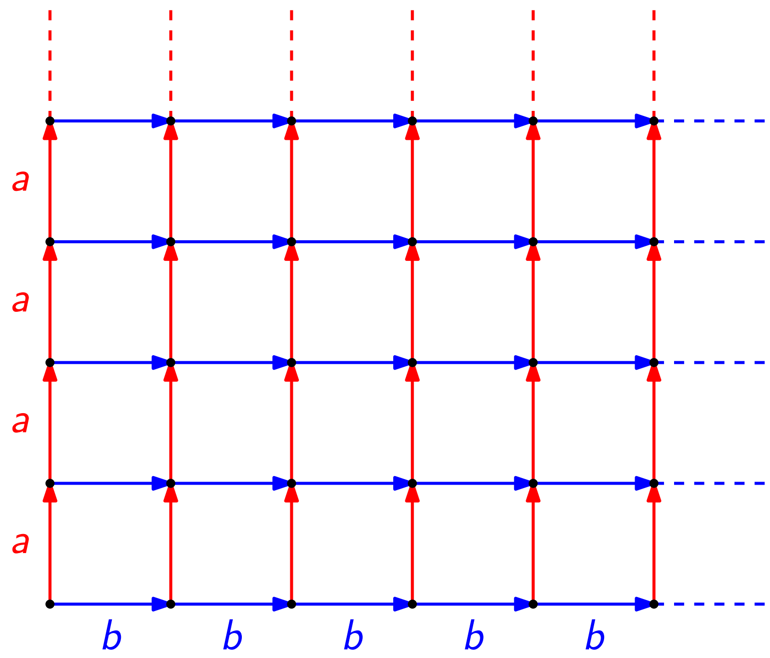
If MSO($D(\mathcal{E})$) is decidable, then MSO($\mathcal{E}$) is decidable

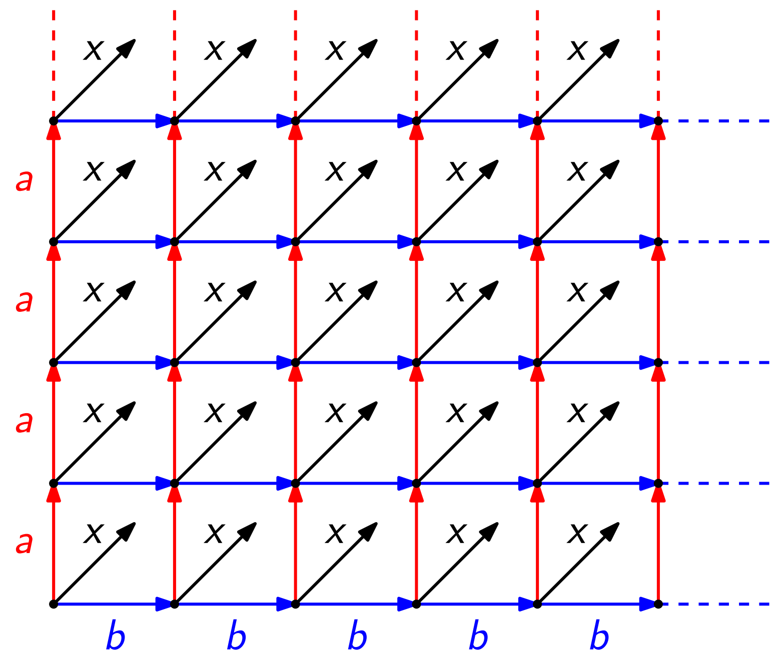
The converse is not true

The hairing of an event structure

Given an event structure $\mathcal{E} = (E, \leq, \#)$, the hairing of $\mathcal{E}$ is $\dot{\mathcal{E}} = (\dot{E}, \dot{\leq}, \dot{\#})$ with:

- $\dot{E} = E \cup E_C$ where $E_C = \{e_c \mid c \in D(\mathcal{E})\}$ is a set of new events
- for any hair event $e_c \in E_C$ and any $e \in \dot{E}$,
  - $e \,\dot{\leq}\, e_c$ if $e \in c$
  - $e \,\dot{\#}\, e_c$ otherwise





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Theorem

If MSO($\dot{\mathcal{E}}$) is decidable, then MSO($D(\mathcal{E})$) is decicable

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Theorem

If MSO($\dot{\mathcal{E}}$) is decidable, then MSO($D(\mathcal{E})$) is decicable

Question

When is MSO($D(\mathcal{E})$) decicable ?

Decidability of MSO($D(\mathcal{E})$)

Theorem

For a regular trace event structure $\mathcal{E} = (E, \leq, \#, \lambda)$, the following are equivalent

(1) *MSO($D(\mathcal{E})$) is decidable*

(2) *$D(\mathcal{E})$ has bounded treewidth*

(3) *the clusters of $D(\mathcal{E})$ have bounded diameter*

(4) *$D(\mathcal{E})$ is a context-free graph*

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(4) *$D(\mathcal{E})$ is a context-free graph*

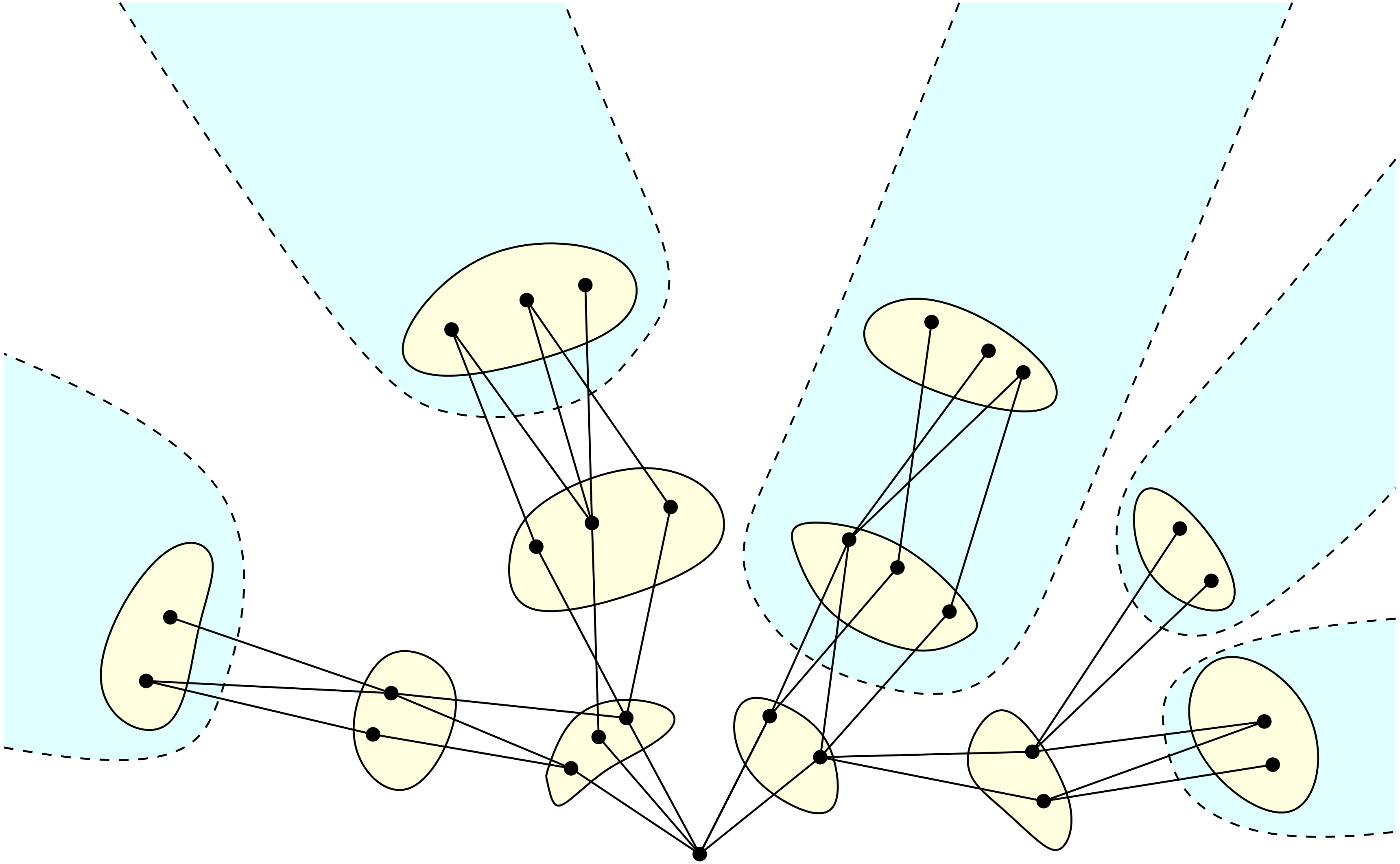
(1) $\Rightarrow$ (2) [Courcelle '94 + Seese '91]

(2) $\Rightarrow$ (3) not today

(3) $\Rightarrow$ (4) [Badouel, Darondeau, and Raoult '99]

(4) $\Rightarrow$ (1) [Müller and Schupp '85]

Clusters, Ends and Context-free Graphs



- Clusters are in yellow, Some ends are in blue
- A graph is context-free if it has finitely many types of ends

Up to hairing, we can work with $D(\mathcal{E})$

Theorem

For a trace regular event structure $\mathcal{E} = (E, \leq, \#, \lambda)$, the following are equivalent

(1) *MSO($D(\mathcal{E})$) is decicable*

(2) *MSO($\dot{\mathcal{E}}$) is decidable*

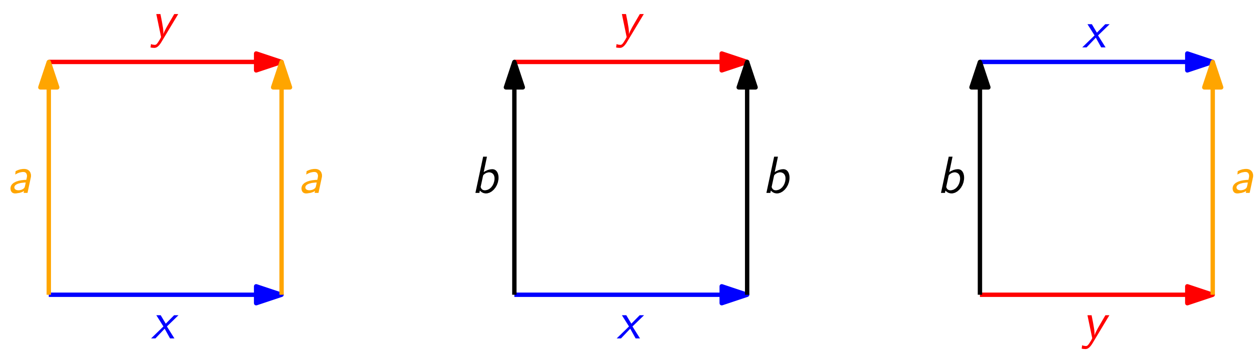
(3) *MSO($D(\dot{\mathcal{E}})$) is decicable*

Question

- Is there a grid-free regular trace event structure $\mathcal{E}$ such that $D(\mathcal{E})$ is not context-free?
- Is there a regular trace event structure $\mathcal{E}$ such that $D(\mathcal{E})$ is hyperbolic and not context-free?

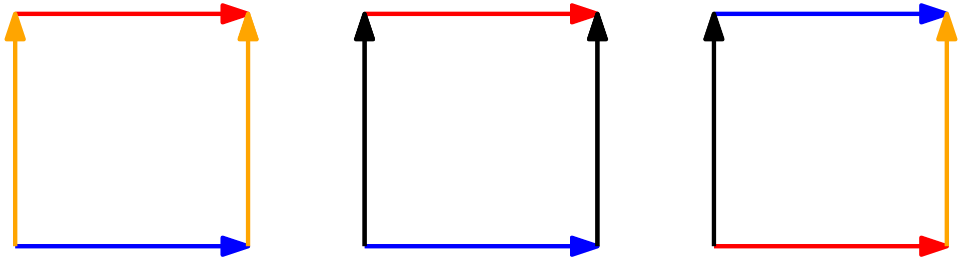
Another complex defined from a set of tiles

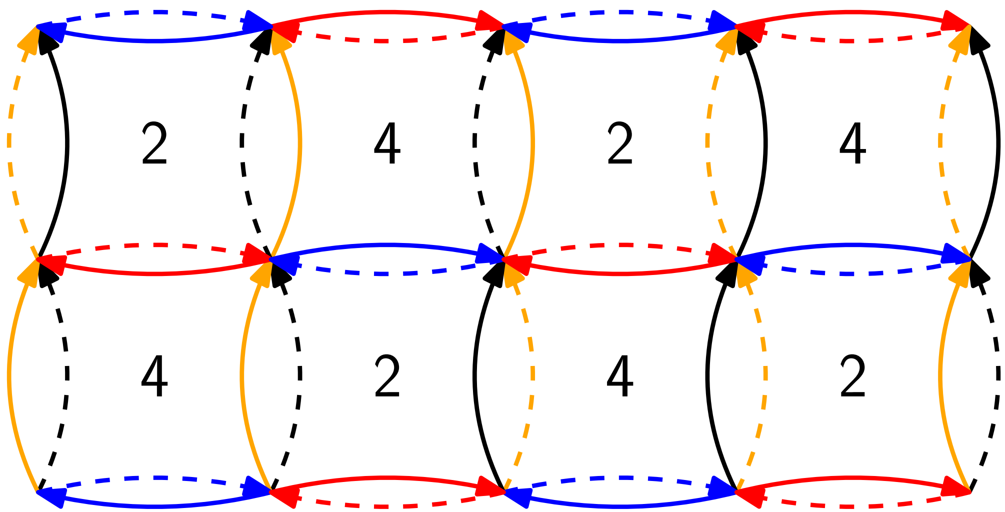
A colored directed NPC complex Z with 1 vertex, 2 “horizontal” edges (x and y), 2 “vertical” edges (a and b), 3 squares:



- Z is a square complex
- Z is directed non positively curved
- Z is not special

Z is virtually special

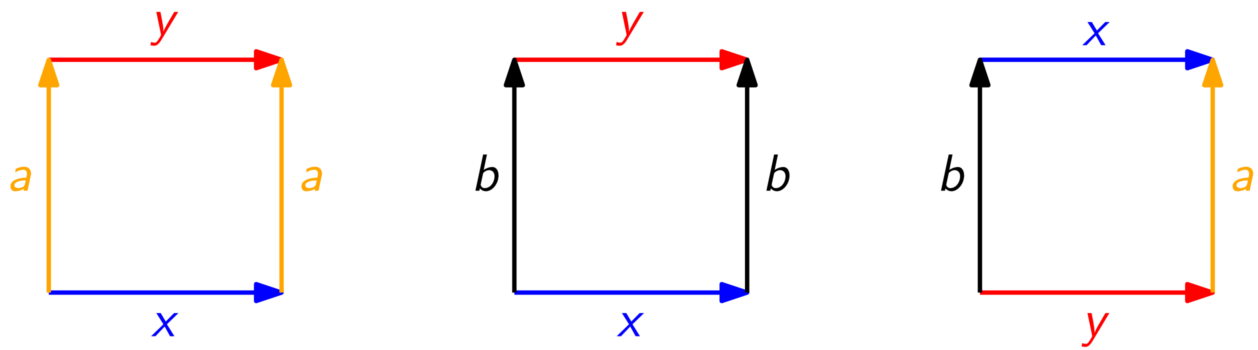




Proposition

$\widetilde{Z}_v$ is the domain of a regular trace event structure $\mathcal{E}_Z$

$\widetilde{Z}_v$ is hyperbolic

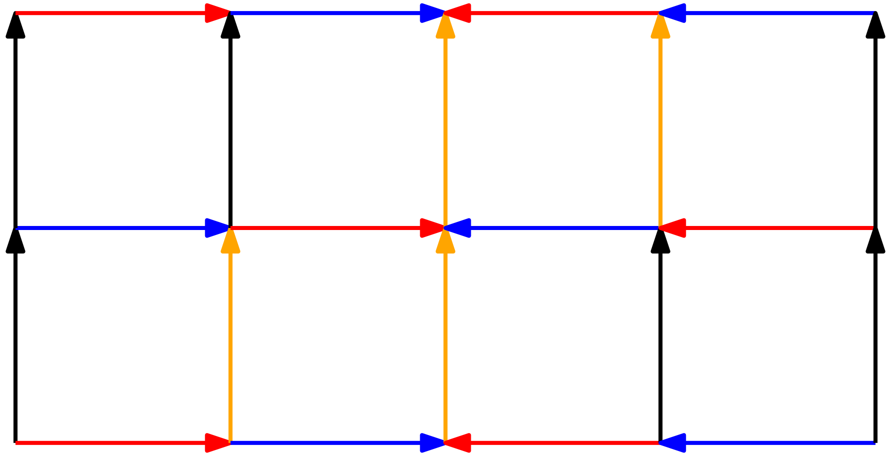


- The tile set defining Z does not tile the plane
- the isometric square grids of $\widetilde{Z}_v$ are bounded
- $\widetilde{Z}_v$ is hyperbolic and thus $\mathcal{E}_Z$ is grid-free

$\widetilde{Z}_v$ is hyperbolic but not $\widetilde{Z}$

Remark

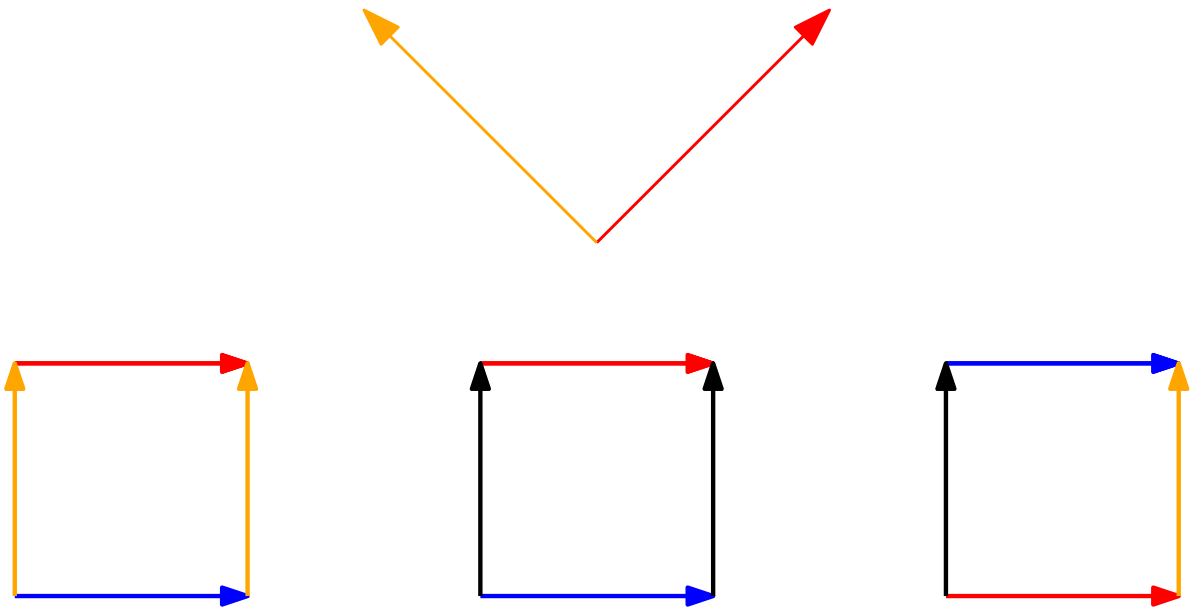
$\widetilde{Z}_v$ is hyperbolic, but $\widetilde{Z}$ is not



Question

When X is a finite NPC complex such that all $\widetilde{X}_v$ are hyperbolic, is X special?

The clusters of $\widetilde{Z}$ are not bounded



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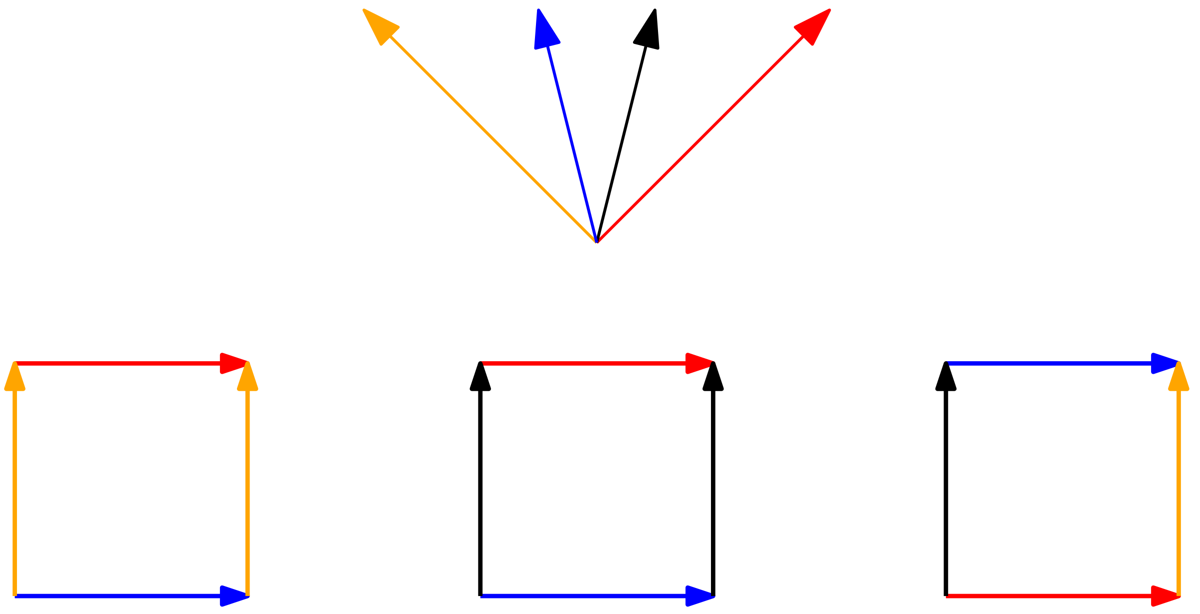

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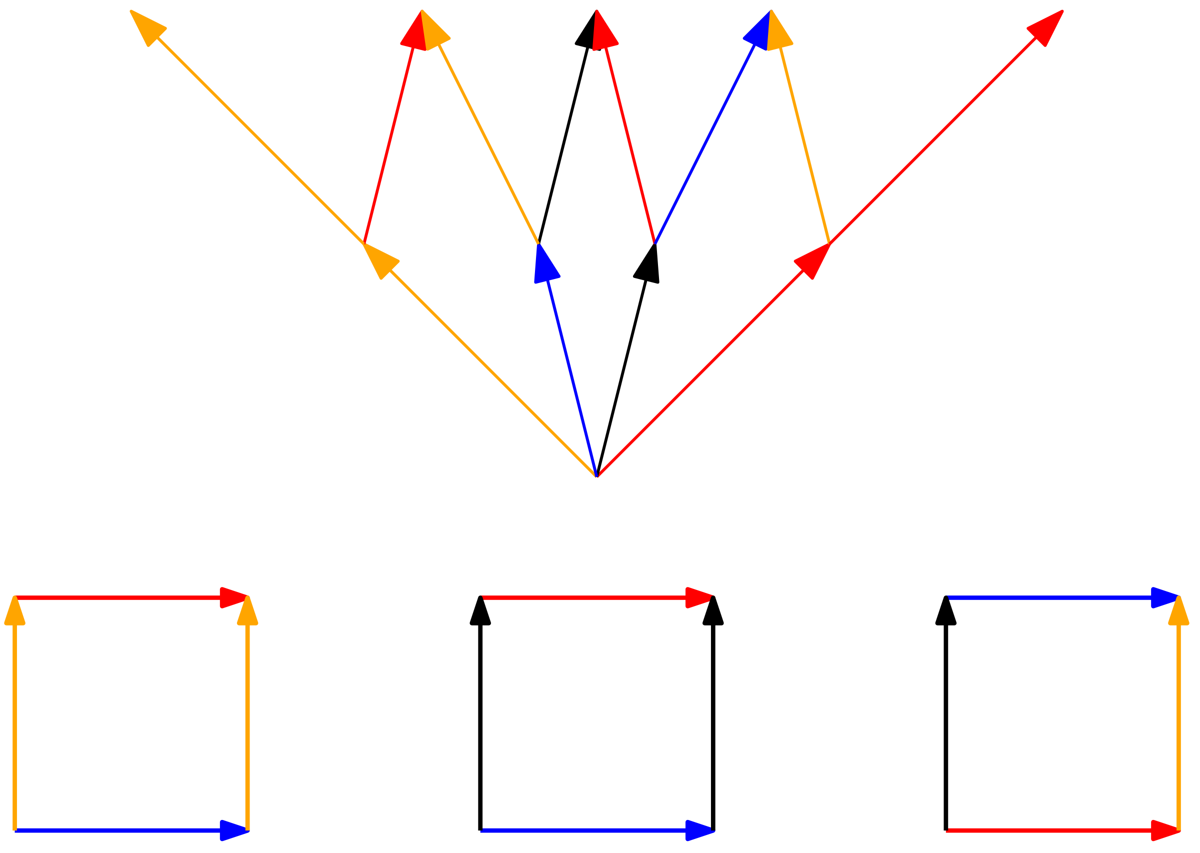

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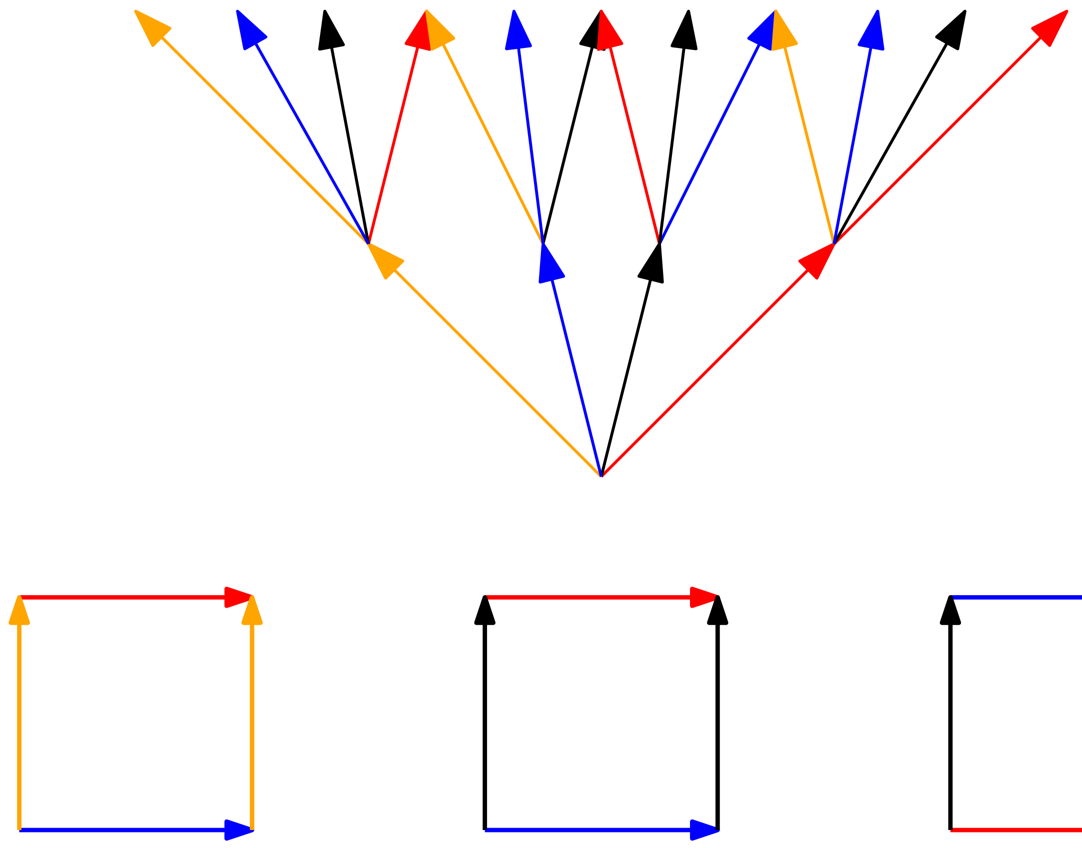

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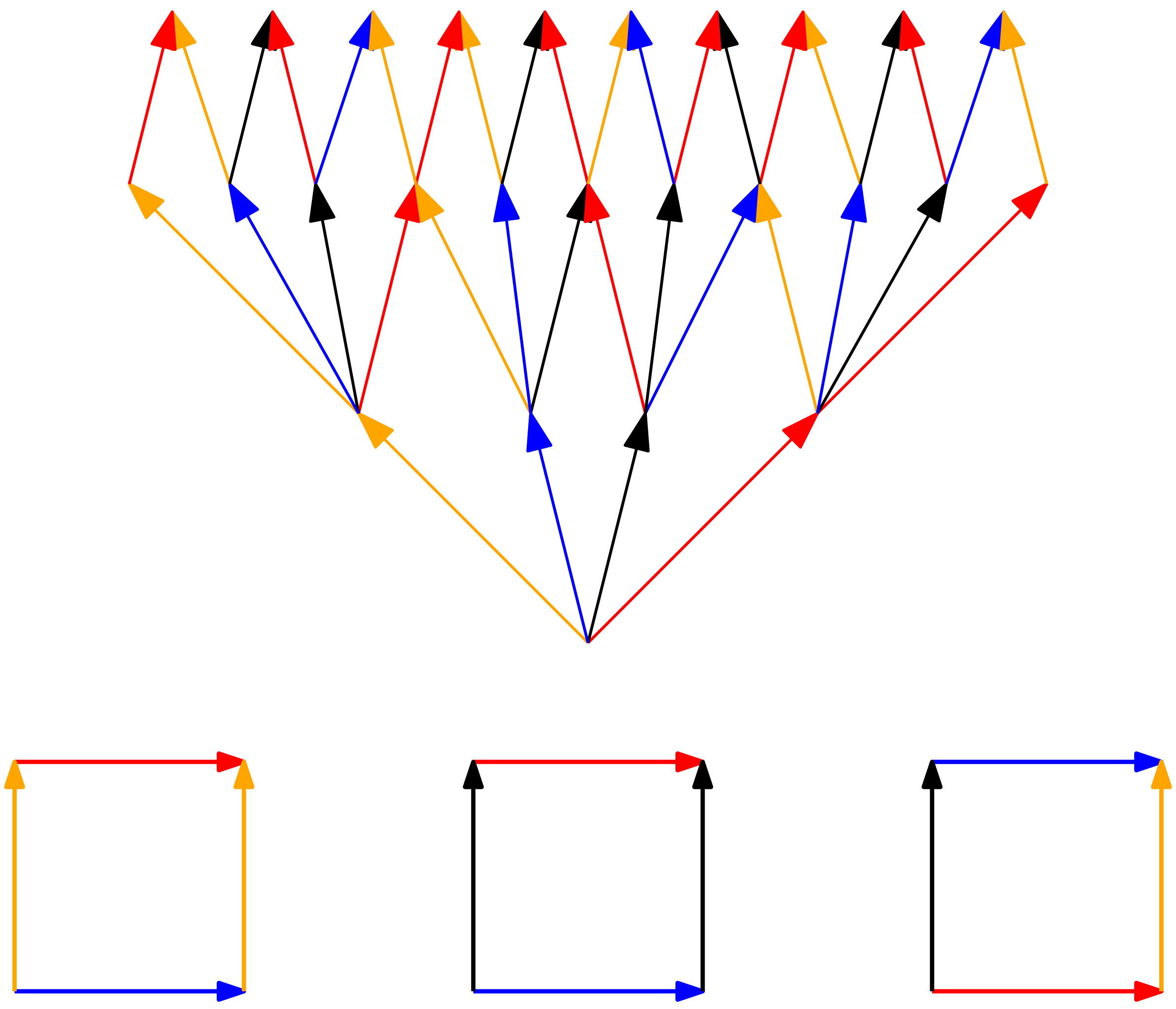

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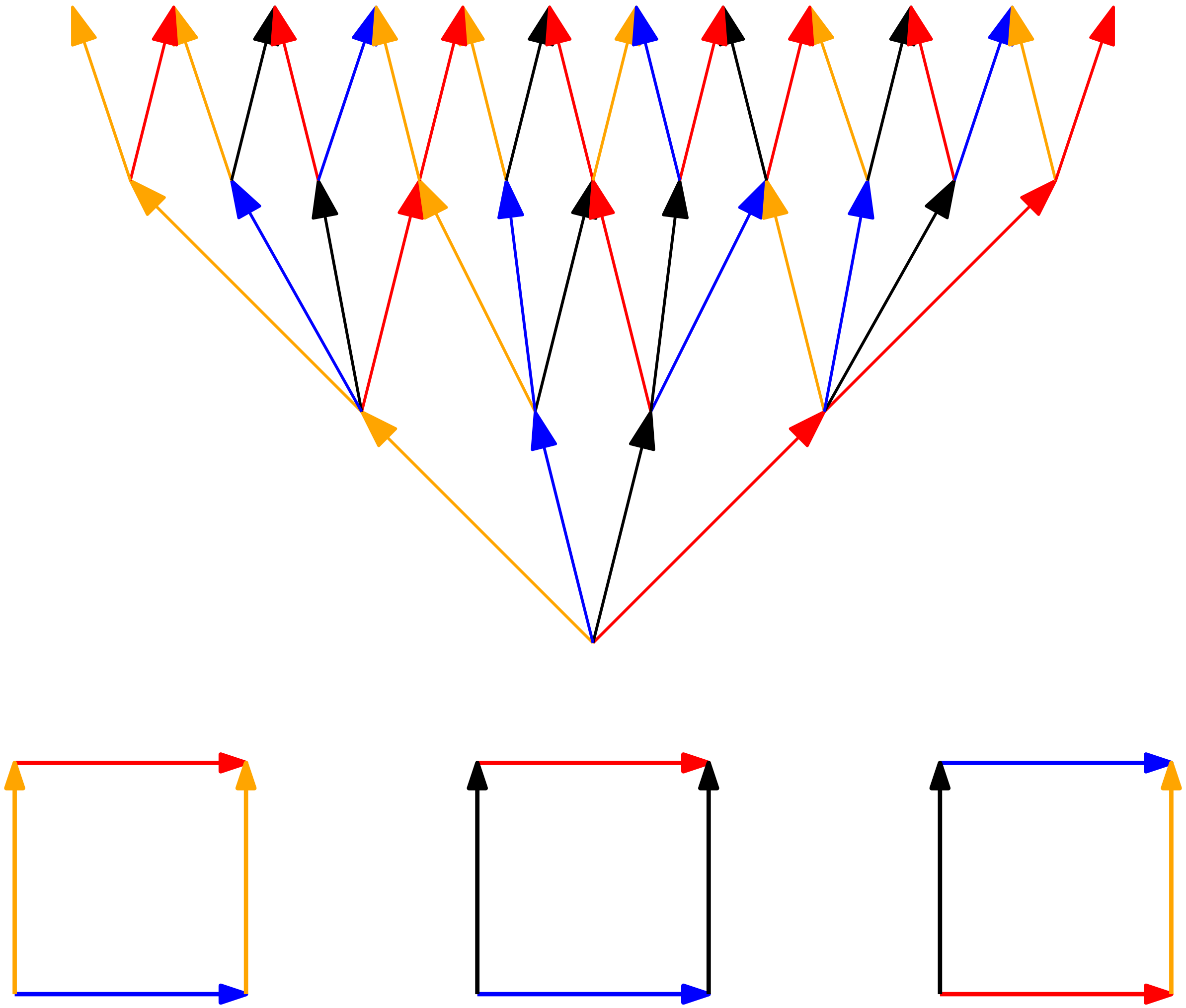


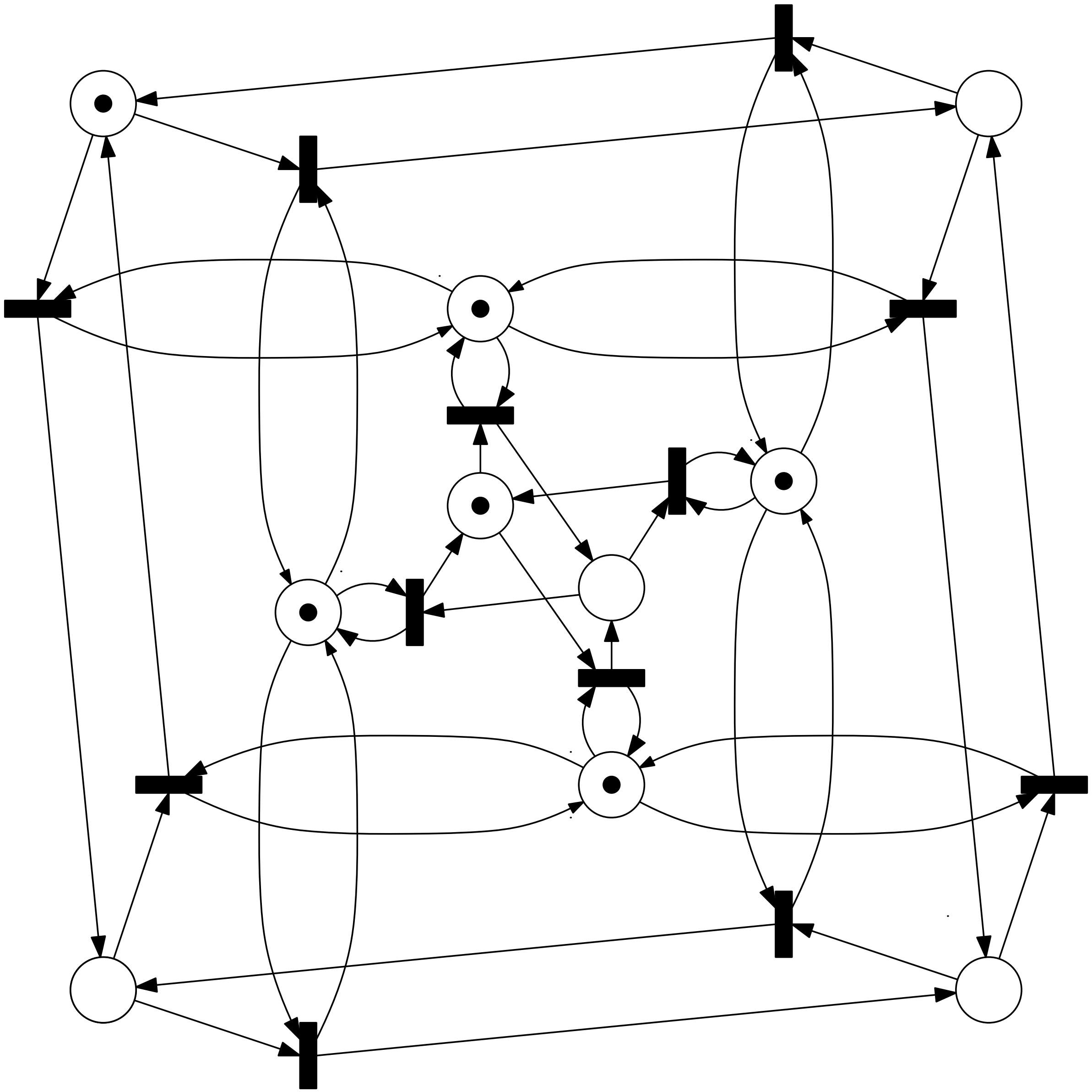
A Counterexample to Thiagarajan's MSO conjecture

- $\widetilde{Z}_v = D(\mathcal{E}_Z)$ and $D(\dot{\mathcal{E}}_Z)$ are hyperbolic
- $\widetilde{Z}_v = D(\mathcal{E}_Z)$ and $D(\dot{\mathcal{E}}_Z)$ are not context-free

Theorem

$\dot{\mathcal{E}}_Z$ is grid-free but MSO($\dot{\mathcal{E}}_Z$) is undecidable

The 1-safe Petri net N_Z



Conclusion

- Negative results,
  - A counter-example to Thiagarajan's regularity conjecture
  - A counter-example to Thiagarajan's MSO conjecture

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  - context-free event domains [Badouel, Darondeau, Raoult '99]
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  - domains obtained from finite NPC complexes with an hyperbolic universal cover
- Questions:
  - Is Thiagarajan's regularity conjecture true for hyperbolic domains?
  - Can we decide if a regular event structure admits a regular nice labelling?

Conclusion

- Nice connections between event structures and NPC complexes
  - CAT(0) cube complexes correspond to event structures
  - finite (virtually) special cube complexes correspond to trace regular event structures
  - Question: Do finite NPC complexes correspond to regular event structures?

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Thank you! Questions?